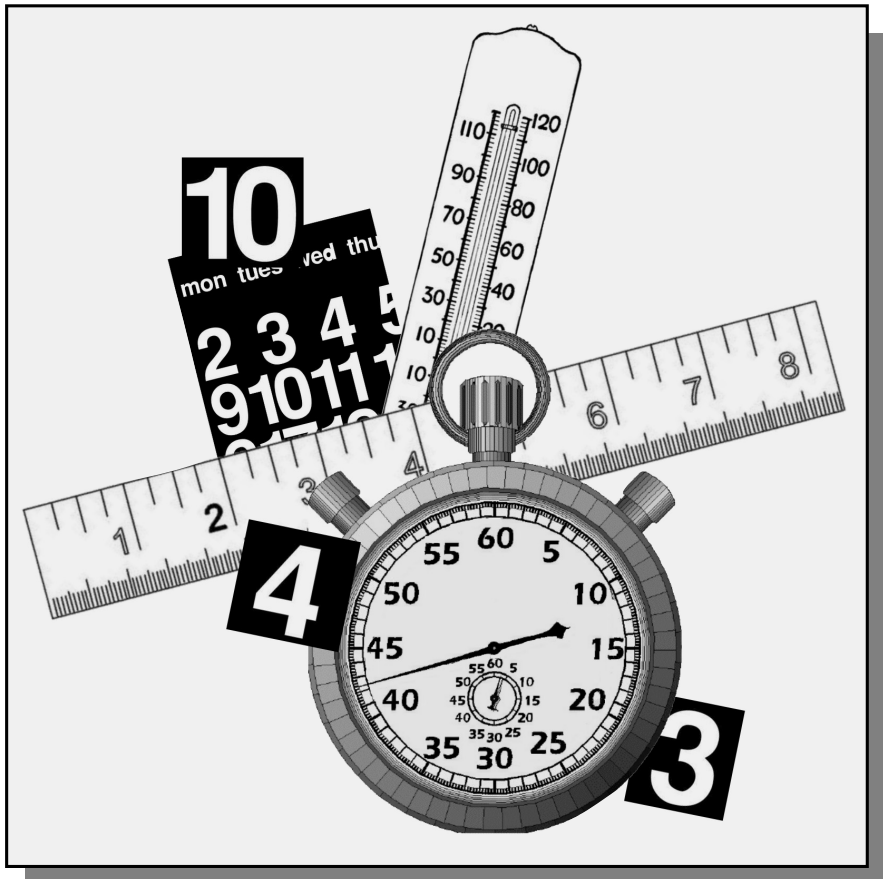
Intermediate Math Skills



Chapter 1

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Intermediate Math Skills

Chapter 1

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CHAPTER 1
TABLE OF CONTENTS

PAGE

SECTION 1

1. CHANCE 1-3
Prediction, Sample Sets, Subsets

2. OUTCOMES 5-7
Probability, Percents

3. LUCK, OR NOT! 9-11
Probability, Fractions

4. SOLVING PROBLEMS 13-17
Conjecture, Patterns

5. PREDICT WITH GRAPHS 19-22
Mutual Funds, Stock Markets, Trends

SECTION 2

6. SIMILAR TRIANGLES 23-26
Corresponding Angles, Congruency, Reflexive Property

7. RATIOS PREDICT 27-28
Similarities, Proportion, Reduction

8. PREDICT AREAS 29-31
Dimensions

PAGE

9. RATIOS AND RECTANGLES 33-35
Relationship of Sides and Area

10. RATIOS SPEAK VOLUMES . . .37-38
Factors of Enlargement and Reduction

SECTION 3

11. PATTERN PROBLEMS 39-41
Exposed Faces, Layers

12. MORE PATTERN PROBLEMS. .43-46
Conjecture, Trial and Error

13. MULTIPLE PREDICTIONS 47-49
Total Probability

14. WHAT ARE THE ODDS? 51-53
Formulas for Computing Odds

15. ENCRYPTION 55-57
Codes, Roman Numerals, Value of π

MATHEMATICS CHARTS 59-61



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"The study of Mathematics cannot be replaced by any other activity that will train and develop man's purely logical faculties to the same level of rationality."

—C. O. Oakley



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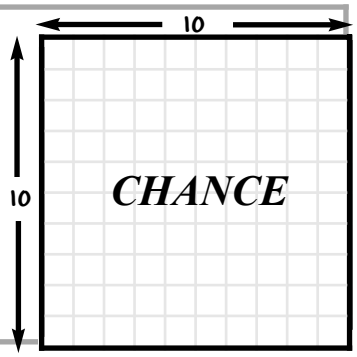
Intermediate Math Skills



Lesson 1

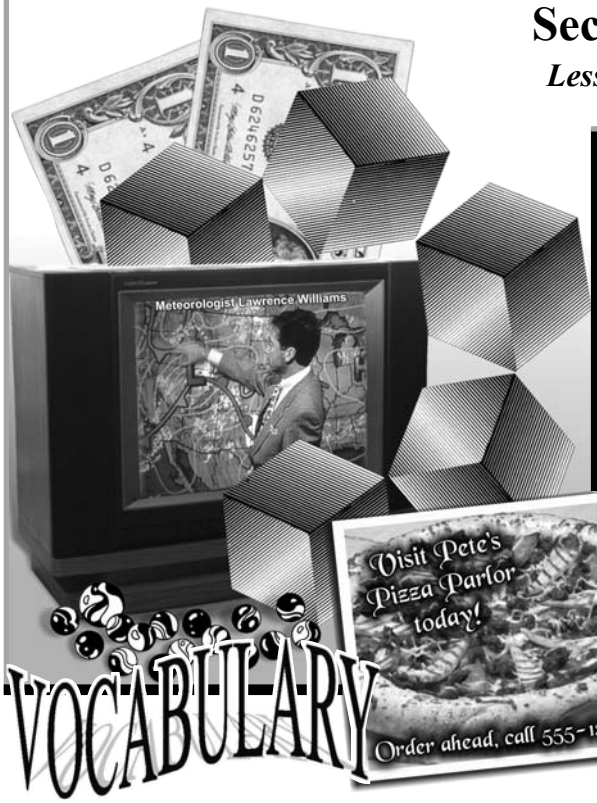
Predictions

Chapter 1



Section 1

Lessons 1-5



Professor K listens to the weather ***predictions*** before he goes to work each day. The meteorologist makes estimations or guesses based on prior occurrences in weather patterns. The weatherman believes that he can predict an ***outcome*** (estimate) based on what he already knows, in this case weather data such as temperatures. Looking at data (a record of previous occurrences) and then predicting an outcome is known in mathematics as *probability*. Sometimes this is referred to as *chance*.

In meteorology (forecasting the weather), the weatherman might say that the chance of rain is 80% today. That means that rain *occurred* 80% or 80 times out of 100 in the past when the weather patterns were the same as today.

Professor K's children play various games that include number cubes, cards and/or spinners. These devices *effect* or *cause* an outcome that ultimately determines a winner. These types of devices are good for games because the probability of a given outcome is never certain, only possible.

In this lesson, Professor K will begin another part of your journey through the land of mathematics to the portion called *probability*. Probability has a language similar to other parts of mathematics, and yet different. Professor K encourages you to study the definitions, vocabulary and examples to familiarize yourself with the terminology of *probability*.

Look at **Example Set 1** as Professor K explains ***sets***, ***sample space***, outcomes, ***events***, ***subsets*** and ***experiments***.

VOCABULARY

Prediction: the telling of an event in advance of it happening; foretelling future events
Outcome: result of an experiment
Set: collection of objects
Sample space: the set containing all possible outcomes
Event: a subset of the sample space
Subset: a set whose members are members of an equal or larger set.
Experiment: something that produces results that can be seen and recorded

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Example Set 1

Professor K once taught a class that consisted of 3 boys and 4 girls. The students' names were Rafael, Michael, Jeffrey, Tina, Nicole, Tammy and Marcy. Determine the following from the students in Professor K's class:

a) the sample space
b) the outcomes if the "experiment" is all the boys in the class.
c) the outcomes if the "experiment" is all the girls in the class.
d) the outcomes if the "experiment" is all the students whose name contains the letter y.



a) sample space:
{Rafael, Michael, Jeffrey, Tina, Nicole, Tammy and Marcy}

a) Professor K writes the names of all the students in the class within braces. This *set* makes up the *sample space*.

b) boys in the class:
{Rafael, Michael, Jeffrey}

b) Professor K knows that the mathematical statement applied to the sample space is like applying heat to a test tube in a chemistry experiment. Both produce observable results. Professor K places the names of all the boys in the class in set symbols {curly brackets}. This is considered an *event*.

c) girls in the class:
{Tina, Nicole, Tammy, Marcy}

c) Professor K places the event of all the girl's names within set symbols.

d) students whose names contain the letter *y*:
{Jeffrey, Marcy, Tammy}

d) Professor K places all the students in the event of names containing the letter *y* in set symbols.

Example Set 2

Professor K visited an automobile dealer and noticed that there were many cars of various types and colors. Professor K considered all the colors of cars on a certain row at the dealership. In this row Professor K saw three white vehicles (w,w,w), two red (r,r), four blue (B,B,B,B) and one black (b) vehicle. Making a math problem from his visit to the automobile dealership, Professor K asked the following questions about:

a) the sample space
b) the outcomes if the "experiment" is all the red vehicles in the row
c) the outcomes if the "experiment" is all the black vehicles in the row
d) the outcomes if the "experiment" is all the white and blue vehicles in the row

a) sample space:
{w, w, w, r, r, B, B, B, B, b}

a) Professor K writes the colors of the vehicles in the row within braces. This set makes up the sample space.

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b) red vehicles in the row:
{r, r}

b) Remember, Professor K knows that the mathematical statement applied to the sample space is like applying heat to a test tube in a chemistry experiment. Both produce observable results. Professor K places the elements of all the red vehicles in the row in set symbols. This is considered an event.

c) black vehicles in the row:
{b}

c) Professor K places the event of all black vehicles within set symbols.

d) white and blue vehicles in the row:
{w, w, w, B, B, B, B}

d) Professor K places all the vehicles in the event of all the white and blue vehicles in the row in set symbols.

Show Professor K your understanding of *probability* by working the activities for this lesson.

LIFE PRINCIPLE



"Learning increases the probability of success."

—Anonymous

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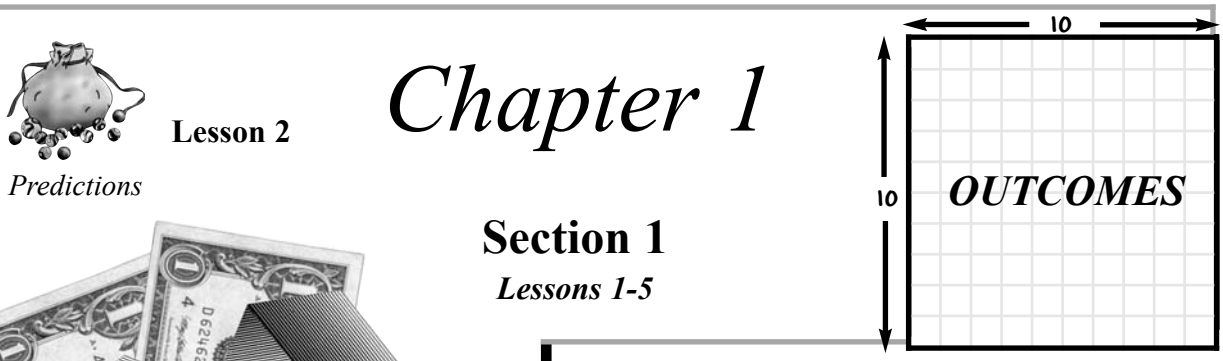


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Chapter 1

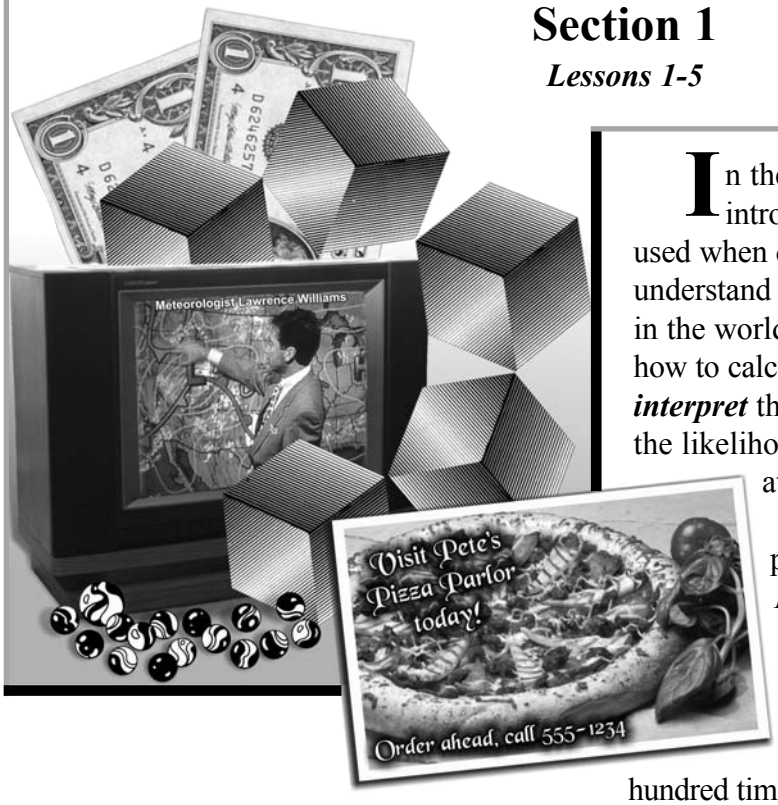
Lesson 2

Predictions



Section 1

Lessons 1-5



In the previous lesson Professor K introduced you to some of the terms used when discussing *probability*. To understand how probability is used most often in the world today, Professor K will describe how to calculate probabilities and discuss or ***interpret*** them. Remember that *probability* is the likelihood that a certain outcome will occur at the end of an "experiment."

Other words used to describe probability are ***chance*** and ***percent***. *Percent* represents the number of actual outcomes for every hundred outcomes that occur. "A 35% chance" means that the outcome mentioned will occur 35 out of one hundred times, such as a 35% chance of rain.

Probability =

$$\frac{\text{the number of elements in the set containing favorable outcomes}}{\text{the number of elements in the sample set}}$$

Professor K reminds you that the *sample set* is the *set of all possible outcomes*. The denominator of the above calculation is the number of *possible outcomes*. Professor K then sets up a ratio of *favorable outcomes* to *possible outcomes* to determine the probability of the event occurring.

Look carefully at the following examples to more fully understand how to determine the probability of a particular outcome of an "experiment."

VOCABULARY

Interpret: to tell what something means
Chance: probability or possibility of something happening
Percent: (%) number of outcomes per hundred

Example Set 1

A bag of marbles contains 50 marbles of which 10 are red, 15 are blue, 10 are green and 15 are yellow. Determine the chance of randomly drawing a red marble from the bag. Then determine the probability of drawing a green marble or a red marble if a marble were picked randomly from the bag.

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To determine these probabilities, Professor K must use the formula for determining probabilities and substitute the *favorable outcome* of picking a red marble from the bag (10) into the numerator and the total number of *possible outcomes* into the denominator (50 marbles in the bag).

- Probability = $^{10}/_{50} = {}^{1}/_{5}$

  Therefore, *one out of five* is the chance of picking a red marble from the bag of fifty marbles.

To calculate the next probability, Professor K must first remember that the bag of marbles contains 10 red marbles, 15 blue marbles, 10 green marbles and 15 yellow marbles. With this in mind, Professor K places the number of *favorable outcomes*, 20 (10 red marbles and 10 green marbles) in the numerator, and 50, the number of possible outcomes (number of marbles in the bag) in the denominator. (Professor K placed the red marble in the previous calculation back in the bag.)

- Probability = $^{20}/_{50} = {}^{2}/_{5}$

  Therefore, the probability of picking a red or a green marble from the bag of marbles is *twenty out of fifty*. Professor K reduces the fraction as shown to obtain a *two in five* chance of picking a red or a green marble from the bag of fifty marbles.

In the next example Professor K explains another aspect of calculating probabilities. Study **Example Set 2** closely as Professor K works another probability problem involving our bag of marbles.

Example Set 2

Our bag of marbles contains 50 marbles, 10 are red, 15 are blue,10 are green and 10 are yellow. Determine the chance of randomly **NOT** picking a red or green marble from the bag of marbles.

The first step Professor K must complete is to add the total number of *favorable outcomes* for each event. (10 red marbles + 10 green marbles = 20 marbles or *20 favorable outcomes*)

Next, since we are looking for the probability of **NOT** picking a red marble or a green marble, Professor K subtracts 20, now the number of *unfavorable* outcomes, from 50, the total number of all outcomes. Professor K obtains an answer of 30 as the number of *favorable outcomes* for this problem.

To determine this probability, Professor K must use the formula for determining probabilities and substitute the favorable outcome (not picking a red or green marble, 30) into the numerator and the total number of possible outcomes into the denominator (50 marbles in the bag).

- Probability = $^{30}/_{50} = {}^{3}/_{5}$

  Therefore after reducing, Professor K determines that *three out of five* is the chance of **NOT** picking a red or green marble from the bag of 50 marbles.

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Show Professor K that you understand calculating probabilities by working the activity for this lesson.

Life Principle



"*Math and life are predictable because of absolutes.*"

—Ronald E. Johnson

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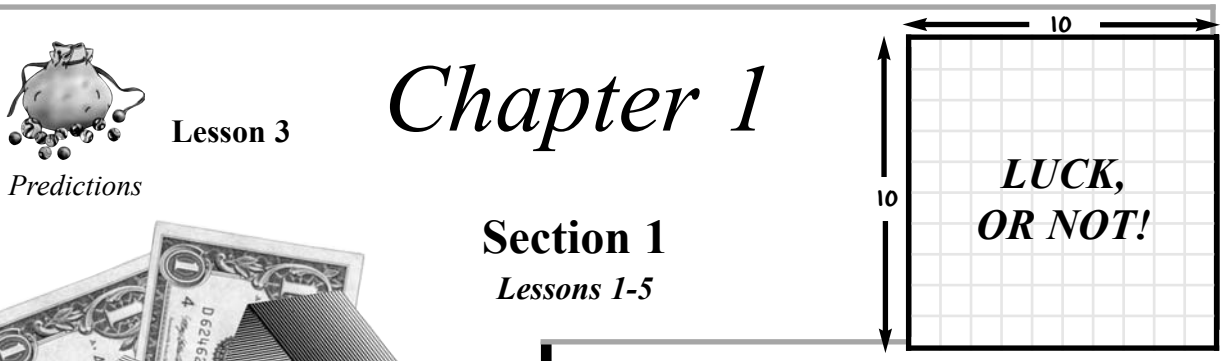


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Chapter 1

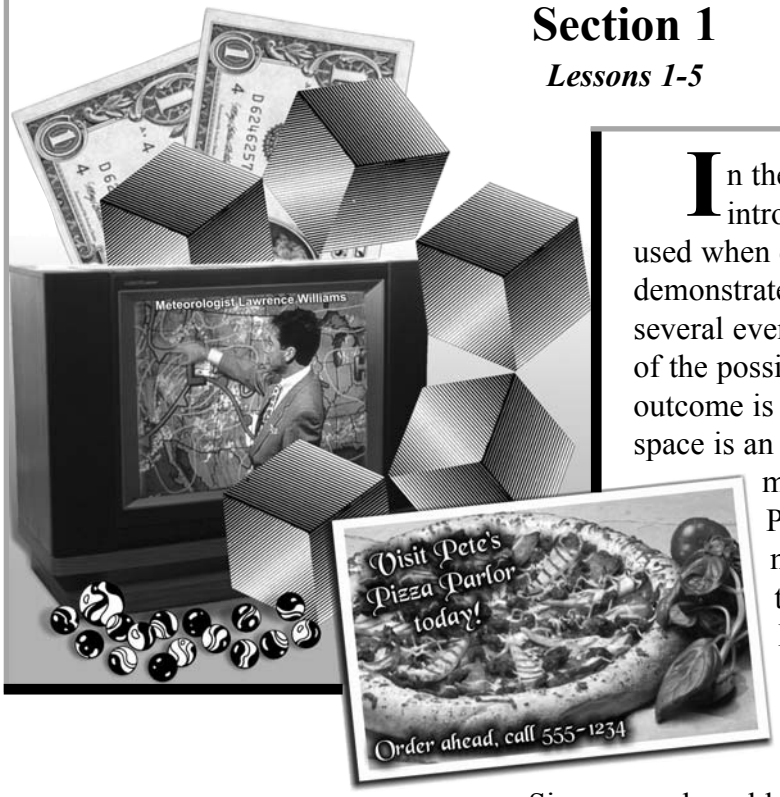
Lesson 3

Predictions



Section 1

Lessons 1-5



In the previous lessons Professor K introduced you to some of the terms used when discussing *probability* and demonstrated how to find the probability of several events occurring. Professor K thought of the possibility of what happens if *no* outcome is possible, or if *all* of the sample space is an outcome. Remember the bag of marbles in the previous lesson? If Professor K pulled all the red marbles out of the bag, then offered the bag to his son, the probability of Professor K's son picking a red marble from the bag would be *zero*.

- Probability (picking a red marble) = $^{0}/_{40} = 0$

Since no red marbles are in the bag, there cannot be an outcome; therefore, a *zero* is placed in the numerator of the probability formula. Professor K points out that *40* is placed in the denominator because he had pulled ten red marbles from the bag of fifty marbles (50 - 10 = 40). Therefore, the number of *possible outcomes* in the sample space is *40*.

It is obvious that when the sample space does not contain the desired outcome then the probability of obtaining that outcome is zero; by following through with the mathematics you can see why. Professor K wants you to consider the complete opposite of *no* possible outcomes. Professor K has some money in his pocket: ten one-dollar bills. Professor K wants to know the probability of pulling a one-dollar bill out of his pocket. Of course, the bill Professor K pulls out will be a one-dollar bill since that was all that is in his pocket, therefore, the probability of pulling a one-dollar bill is one, *1*.

- Probability (picking a one-dollar bill) = $^{10}/_{10} = 1$

VOCABULARY

Element: one of the parts that make up the whole
Translates: change; to tell what something means

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Notice that the number of possible desired outcomes is *10* and that number is placed in the numerator. The number of ***elements*** in the sample space is also *10* and that number is placed in the denominator. Professor K reminds you that when the numerator and denominator of a fraction are the same, the value of the fraction is *1*. A probability of *1* ***translates*** into an outcome happening 100% of the time.

Look at the examples below as Professor K shows you more problems in probability.

Example Set 1

A six-sided number cube is labeled on each side with the numbers 1, 2, 3, 4, 5, and 6. Determine the chance in a given throw, of the cube landing with the 5 showing in the up position.



a) Professor K notices that the sample space contains six possible outcomes; 1, 2, 3, 4, 5, or 6. There is only one chance to produce an outcome of the number five. Professor K places the number *1* in the numerator of the probability formula and *6*, the number of possible outcomes in the sample space in the denominator.

   - Probability = $^1/_6$

     Therefore, *one out of six* is the chance of the number cube landing with the *5* in the up position.

b) Determine the chance that the number in the up position is *not* a 5. Professor K realizes that there are five outcomes from the sample space that are not 5. Professor K therefore, places the number *5* in the numerator in the probability formula to determine the probability of not rolling a *5*. Professor K then places the number of possible outcomes, *6*, in the denominator.

   - Probability = $^5/_6$

c) Determine the chance of rolling a *7* with the number cube. Professor K looks at all the possible outcomes in the sample space and determines that the number *7* is not in the set. Therefore, the probability of rolling a *7* is *0*, as shown in the equation below.

   - Probability = $^0/_6 = 0$

In the next example Professor K explains another aspect of calculating probabilities. Study **Example Set 2** closely as Professor K works another probability problem involving a twelve-sided number cube.

Example Set 2

A number cube consisting of twelve sides is labeled with one of the following letters on each side: A, A, B, C, D, D, D, E, F, G, G, H. Determine the probability of each letter ending in the up position after rolling the cube.

Professor K first determines the value of the denominator of the probability formula to be *12* since there are twelve sides to the number cube and therefore, twelve possible outcomes. Next, Professor K counts the number of times each letter appears in the sample space and

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places that count in the numerator of the probability formula to calculate the probability of rolling that letter. Follow Professor K's calculations below as he calculates each of these probabilities.

Number of *A*s = 2	Probability (rolling A) = $^2/_{12} = {}^1/_6$
Number of *B*s = 1	Probability (rolling B) = $^1/_{12}$
Number of *C*s = 1	Probability (rolling C) = $^1/_{12}$
Number of *D*s = 3	Probability (rolling D) = $^3/_{12} = {}^1/_4$
Number of *E*s = 1	Probability (rolling E) = $^1/_{12}$
Number of *F*s = 1	Probability (rolling F) = $^1/_{12}$
Number of *G*s = 2	Probability (rolling G) = $^2/_{12} = {}^1/_6$
Number of *H*s = 1	Probability (rolling H) = $^1/_{12}$

After determining the probabilities in **Example Set 2,** Professor K can look at the results and answer the questions in **Example Set 3**.

Example Set 3

1. Which letter is most likely to appear on top?	1. The *D* because the value of the fraction, $^1/_4$, representing the probability that is the largest, $^1/_4 > {}^1/_6 > {}^1/_{12}$.
2. Which letters have the same likelihood of appearing on top?	2. The *B*, *C*, *E*, *F* and *H* all have an equal chance of appearing on top because the values of the fractions representing their probability are the same, $^1/_{12}$. The *A* and *G* have an equal chance of appearing on top because the values of the fractions representing their probability are the same, $^1/_6$.
3. Which is more likely to be the result of rolling the number cube: an A, B or E, or to roll a D?	3. In this case Professor K adds the probability of rolling an *A*, *B* and *E* together to get: $^1/_6 + {}^1/_{12} + {}^1/_{12} = {}^4/_{12} = {}^1/_3$. Since $^1/_3$ is greater than $^1/_4$, it is more likely to roll an A, B or E than it is to roll a D.

Professor K wants you to realize that many questions can be asked and answered as in **Example Set 3**, but you too can answer them if you will take your time and think through the question that is asked. Show Professor K that you can work these types of probability problems by working the activities for this lesson.

LIFE PRINCIPLE



"A clean heart translates into a clean life."

—Anonymous

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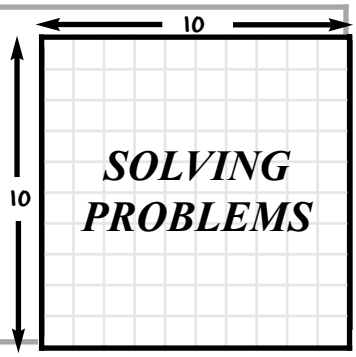
Intermediate Math Skills



Lesson 4

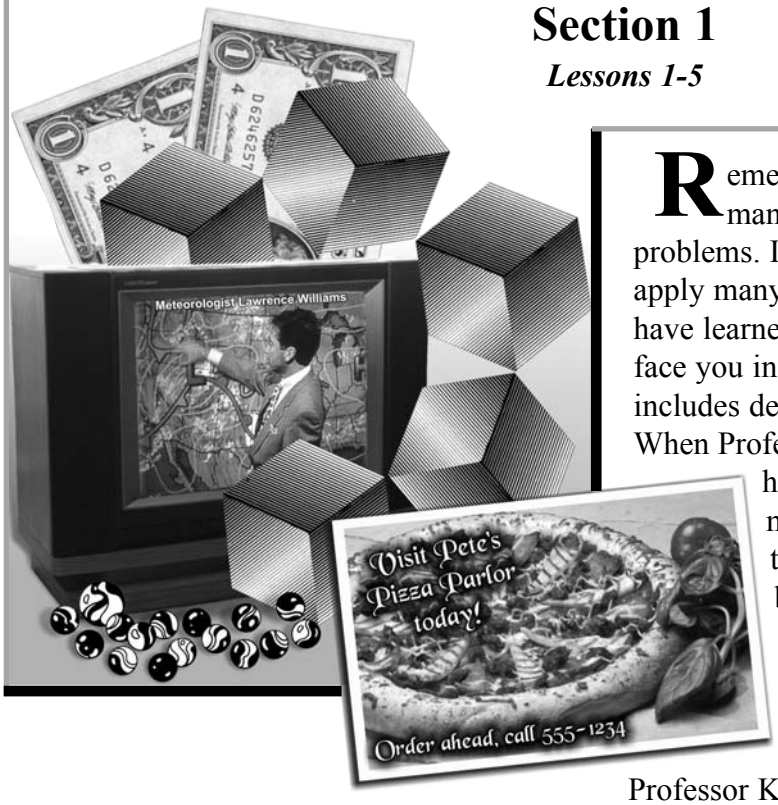
Predictions

Chapter 1



Section 1

Lessons 1-5



Remember that Professor K believes many aspects of life are like word problems. In this chapter, Professor K will apply many of the mathematical concepts you have learned to answer questions that could face you in your daily life. Solving a problem includes determining or predicting an outcome. When Professor K's car stops running suddenly, he begins to solve the problem by making a ***conjecture*** as to what is the problem. Professor K then begins to work through the facts surrounding the conjecture and works out a solution. Probability is one of the tools you can use to predict an outcome. In this chapter Professor K will show you how to use graphs and observable ***patterns*** in number groups to make predictions, as well as more examples of probability situations. To begin this study, Professor K will make predictions based on observable patterns of numbers. Look at the following steps that Professor K uses to solve problems. These steps are similar to the steps you learned earlier for solving stated problems. Professor K found these steps in a book entitled *How To Solve It*, written by a man named George Polya.

Steps to How To Solve It

Step 1	You have to understand the problem.	<u>Understanding the Problem</u> a) What is going on in the problem? What is wanted? Is it solvable? b) Draw a picture. Use the correct formulas.

VOCABULARY

Conjecture: conclusion usually reached by guesswork; a theory
Pattern: something repeated
t_1, t_2, t_3, t_4… t_n: notation used to signify the value of a specific term in a series of numbers; subscript is the number of the term, *n*

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		Devising a Plan
Step 2	Find a connection between the data and the solution. Obtain a plan to solve the problem.	a) Have you seen this type of problem before? b) Do you know a similar problem? c) Can you use the same methods? d) Can you restate the problem? e) Did you use all the data?
		Carrying Out the Plan
Step 3	Carry out your plan.	a) Carry out your plan, checking each step for accuracy.
		Looking Back
Step 4	Examine the solution obtained.	a) Can you check the answer? b) Can you derive the result differently? c) Can you use the result, or the method, for another problem?

Look at **Example Set 1** as Professor K shows you how to make predictions using Polya's steps to solving a problem.

Example Set 1

Find the next term in the series of numbers given, and then the 100th term.

1, 3, 5, 7, 9 . . .

Understanding the problem may mean you must reread the instructions, the problem and the data presented, more than once. This problem is to find the sixth number in the series of numbers.

Step 1	1, 3, 5, 7, 9, 11 . . .	Professor K notices that the numbers are odd, and therefore the next number would be *11*. However, this fact does not help to determine the 100th term without writing the ninety-nine preceding terms.
Step 2	9 + 2 = 11	Professor K notices that adding *2* to the previous term will find the next term, but he needs a way to find the first term. Professor K needs to find a formula or way to calculate every term including the first. Professor K attempts to relate the value of a term to the term number.



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Step 3

first term = $t_1 = 1$
second term = $t_2 = 3$
third term = $t_3 = 5$
fourth term = $t_4 = 7$
fifth term = $t_5 = 9$

Professor K writes each term in term number notation along with its value. Then he attempts to find a way to use the term notation as a variable in a formula that can be used to produce the value of any term just by knowing the term number.

Step 4

$n + 2$, gives the value of all terms except the first; therefore cannot be used.
$2 \cdot n$ yields: $t_1 = 2$, $t_2 = 4$, $t_3 = 6$; generating even numbers; therefore cannot be used.
$2 \cdot n - 1$ yields:
first term = $t_1 = 2 \cdot 1 - 1 = 1$
second term = $t_2 = 2 \cdot 2 - 1 = 4 - 1 = 3$
third term = $t_3 = 2 \cdot 3 - 1 = 6 - 1 = 5$
fourth term = $t_4 = 2 \cdot 4 - 1 = 8 - 1 = 7$
fifth term = $t_5 = 2 \cdot 5 - 1 = 10 - 1 = 9$

Professor K uses the letter, n, to represent the number of the term. He then writes mathematical expressions to attempt to produce the value of each term.

Step 5

100th term = $t_{100} = 2 \cdot 100 - 1 = 200 - 1 = 199$

Looking at the calculations in Step 3, Professor K believes that he has found the correct formula for calculating the value of a given term. Therefore, Professor K calculates the 100th term by placing 100 in the formula. Professor K then calculates the value of the hundredth term.

Example Set 2

Find the tenth term of the series of numbers. Find the formula to produce the n^{th} term.
0, 2, 4, 6, 8 . . .

Step 1

0, 2, 4, 6, 8, 10, 12,
14, 16, 18

Professor K notices that the numbers are even, and the tenth term is easy to determine just by carrying the even number count forward five more times: *10*, *12*, *14*, *16*, *18*. Therefore, the tenth number would be *18*. However, this fact does not help to determine the n^{th} term without writing all of the preceding terms. Professor K simply counts forward by twos to calculate the tenth term. He uses this to help check his formula for the n^{th} term.

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Step 2	$n + 2 =$	Professor K notices that adding *2* to the previous term will find the next term, but he needs a way to find the first term. Professor K needs to find a formula or way to calculate every term including the first. Professor K attempts to relate the value of a term to the term number.
Step 3	$t_1 = 0$, $t_2 = 2$, $t_3 = 4$, $t_4 = 6$, $t_5 = 8$	Professor K writes each term in term number notation along with its value. Then he attempts to find a way to use the term notation as a variable in a formula that can be used to produce the value of any term just by knowing the term number.
Step 4	$n + 2$, gives value of all terms except first; therefore cannot be used. $2 \bullet n$ yields: $t_1 = 2$, $t_2 = 4$, $t_3 = 6$; generating even numbers, but the first term of zero is missing; therefore cannot be used. $2(n - 1)$ yields: first term = $t_1 = 2(1 - 1) = 2(0) = 0$ second term = $t_2 = 2(2 - 1) = 2(1) = 2$ third term = $t_3 = 2(3 - 1) = 2(2) = 4$ fourth term = $t_4 = 2(4 - 1) = 2(3) = 6$ fifth term = $t_5 = 2(5 - 1) = 2(4) = 8$ tenth term = $t_{10} = 2(10 - 1) = 2(9) = 18$	Professor K attempts several equations to find the one that will generate any term desired.
Step 5	$t_{11} = 2(11 - 1) = 2(10) = 20$	Professor K reads the instructions and the problem and realizes that it is similar to the previous example. Therefore, he understands the basic principle for calculating the solution to the problem will be similar. Professor K feels confident in his generalized formula, $2(n - 1)$. Professor K checks his formula by calculating the next, eleventh, term. Professor K sees that the eleventh term is the same using the formula and counting forward by two, so he believes that he has found the correct formula.

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Professor K wants you to notice that in both examples he followed Polya's steps to solve a problem by understanding what was given and what was asked. Professor K devised a plan by working the problem several ways using all the information, data, given. Professor K then carried out his plans by making attempts, checking results, making adjustments, then finding the correct formula. Professor K evaluated the solution by calculating another term whose value could be checked easily.

Show Professor K you understand how to make predictions based on observable patterns of numbers by working the activities for this lesson.

Life Principle



"Each problem that I solved became a rule which served afterwards to solve other problems."

—René Decartes

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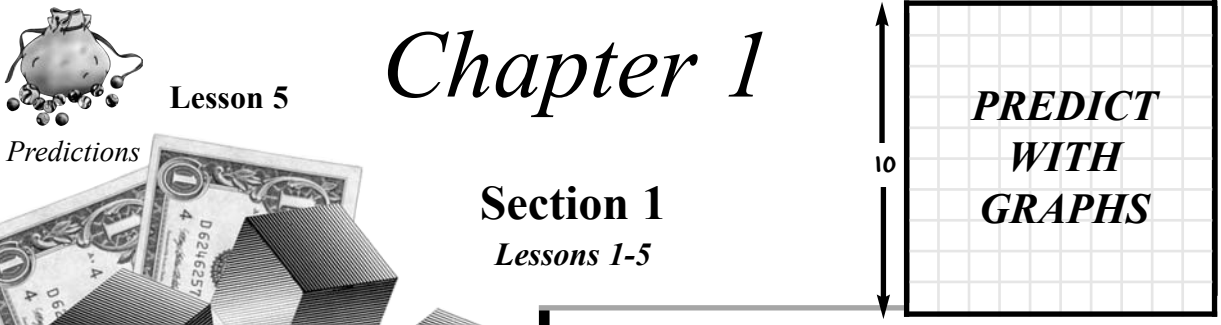
Chapter 1

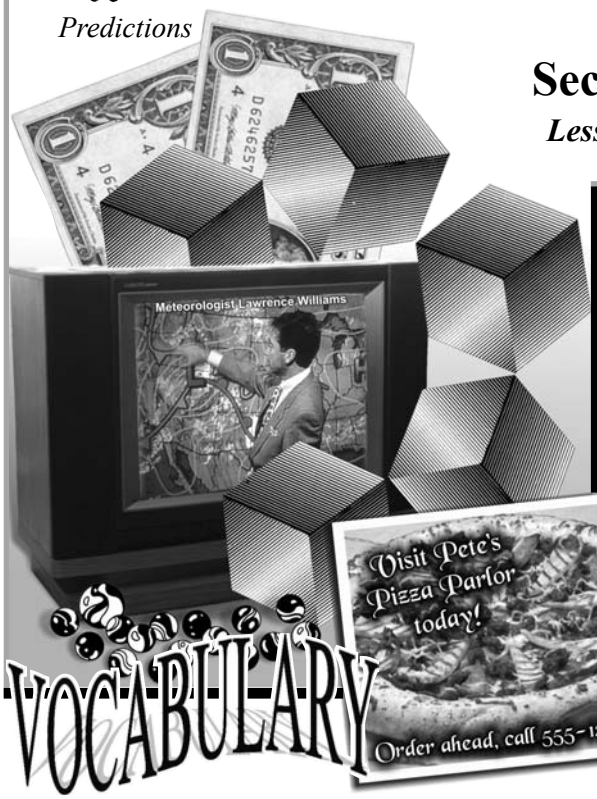
Lesson 5

Predictions

Section 1

Lessons 1-5





VOCABULARY

Mutual funds: an investment company that uses members capital (money) to buy a diverse group of stocks from other companies
Stock market: place where stocks are bought and sold
Commodity: something used in trade
Stocks: equal fractional value of certain companies, purchasable by private citizens who then own a share in that company (which shares they may sell)
Analysts: persons who examine issues in careful detail as to causes, key factors and possible or probable results
Trend: to move in a certain direction; an overall pattern of events

Professor K invests money in ***mutual funds*** that he can use when he retires. The person that manages mutual funds like the one in which Professor K invests watches the ***stock market*** and ***commodity*** market for patterns. ***Stock*** and commodity prices are listed in newspapers and on the internet daily. The wise manager looks at the market reports and searches for patterns and then buys or sells stocks or commodities accordingly. The quickest method that stock market ***analysts*** use to look for patterns is *graphs*. Stock price averages are shown on line graphs. Most businesses and corporations use bar graphs and pie graphs to show income and expenses as well as profits and losses. Production and sales are also usually shown with bar and pie graphs.



Professor K

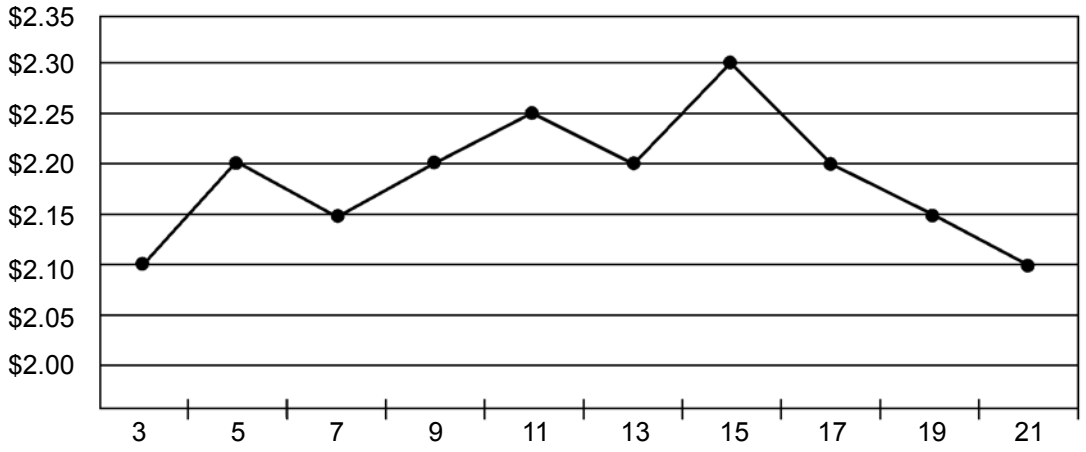
Stock market analysts make predictions as to the future of certain stocks or commodities by using the information contained on the line graphs. Businesses and corporations use the pie and bar graphs to make predictions relating to production and labor.

In this lesson, Professor K will make predictions based on observable patterns observed from graphs. Follow the steps that Professor K uses to make predictions using various graphs.

Look at **Example Set 1** as Professor K shows you how to make predictions using a line graph.

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Example Set 1



Natural Gas Prices per cubic foot

During a given month Professor K watched natural gas prices (as shown in the graph above). Looking at the graph, Professor K attempted to make a prediction as to what the price would be the next day for natural gas. (Prices are plotted along the y-axis and the date along the x-axis.)

Step 1 Professor K looks at the trend of natural gas prices for the days 3-15. This ***trend*** overall is *upward* or the price of natural gas increased from $2.10 per cu ft to $2.30 per cu ft.

Step 2 Professor K looks at the trend of natural gas prices for the days 15-21. This trend overall is *downward* or the price of natural gas decreased.

Step 3 When looking at the entire graph, Professor K notices that only twice did the price change more than five cents over two days. Eight out of ten times the price only changed by five cents. Professor K concludes that the price on the 23rd would only change another five cents.

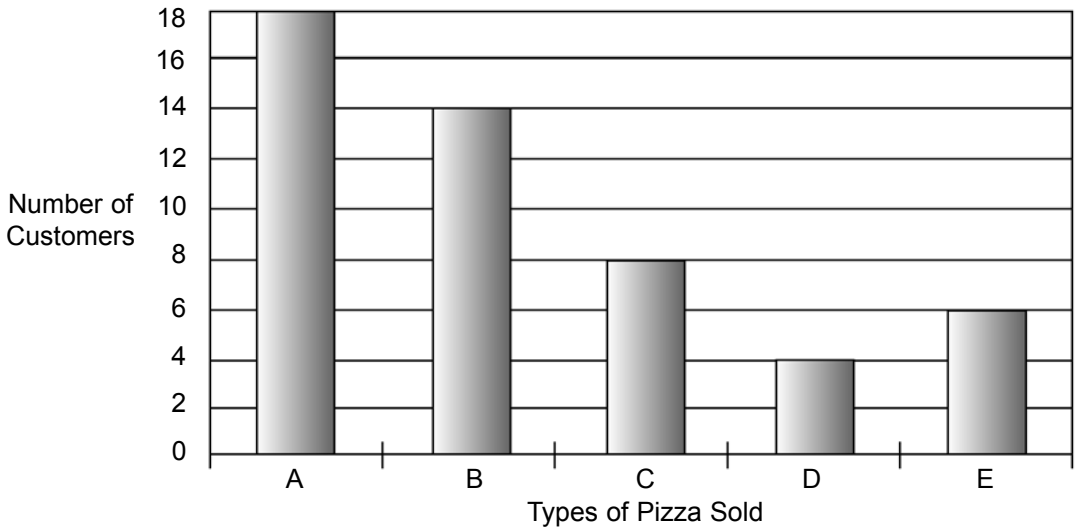
Step 4 Professor K looks at the downward trend of the previous six days and predicts that the price of natural gas on the 23rd would continue downward.

Step 5 Professor K combines his conclusions from Step 3 and Step 4 to predict that the price of natural gas on the 23rd will be $2.05.



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Example Set 2



A friend of Professor K owns a pizza parlor. During the "lunch run" the chart above shows the frequency with which patrons of the pizza parlor ordered various types of pizzas (*pepperoni, hamburger, sausage, supreme, other*). Professor K's friend handed Professor K the chart and then asked him several questions to see if Professor K could figure them out.



Question	**Answer**
1. If more *pepperoni* were sold than any other pizza, which column represents the number of *pepperoni* pizzas ordered?	A
2. How many *pepperoni* pizzas were ordered?	18
3. If orders of *supreme* pizza were two less than the orders of *other* pizzas and four less than the orders of *sausage*, which columns represent the three different kinds of pizza?	Professor K looks at the chart and realizes that column D is the only column that is two less than one column and four less than another column at the same time. This means that column D represents the number of *supreme* pizzas ordered. Column E would represent *other* pizzas and column C would represent the number of *sausage* pizzas ordered. Looking at the chart Professor K determines that the patrons ordered 4 *supreme* pizzas, 6 *other* pizzas and 8 *sausage* pizzas.
3. How many *hamburger* pizzas were sold?	14, because column B is the only column remaining to represent *hamburger* pizzas.

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4. By looking at the chart, should Professor K's friend be too worried about the lunch run the next day if he did not have any more mushrooms.

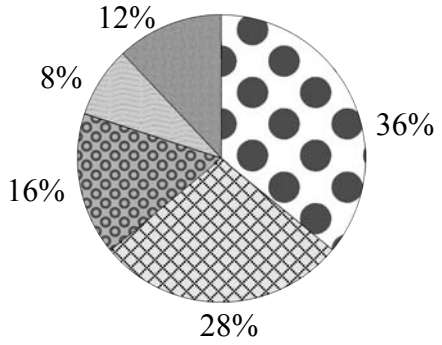
No, because if you assume that the lunch run everyday is nearly the same, then the chart shows that *pepperoni*, *hamburger*, and *sausage* make up most of the pizza sales.

Professor K wants you to notice that in **Example Set 2** he made an assumption based on the information given. That information was not as accurate as it could otherwise be, if Professor K could have seen data from the previous week, month or even year. The more data that is available, the better the prediction.

Example Set 3

Since Professor K's friend owned a pizza parlor, Professor K thought that it would be fun to change the bar graph in **Example Set 2** to a "pizza" graph. Professor K calculated the percentage of each type of pizza sold by dividing the numbers for each pizza from **Example Set 2** by the total of all the pizzas sold, 50.

Pepperoni = $^{18}/_{50} = {}^{9}/_{25} = .36 = 36\%$
Hamburger = $^{14}/_{50} = {}^{7}/_{25} = .28 = 28\%$
Sausage = $^{8}/_{50} = {}^{4}/_{25} = .16 = 16\ \%$
Supreme = $^{4}/_{50} = {}^{2}/_{25} = .08 = 8\%$
Other = $^{6}/_{50} = {}^{3}/_{25} = .12 = 12\%$



Professor K likes graphs because the data can be interpreted more quickly. Remember that a graph is a pictorial representation of the data. It has been said that a picture is worth a thousand words. By looking at graphs such as this pie graph, you do not have to know the numbers to know that if the polka-dotted area is pepperoni, then more pepperoni was sold than any other. At a glance it is also obvious that supreme pizzas are the smallest area.

Show Professor K you understand how to make predictions based on observable patterns of numbers by working the activities for this lesson.

LIFE PRINCIPLE



"A picture is worth a thousand words."

—Anonymous

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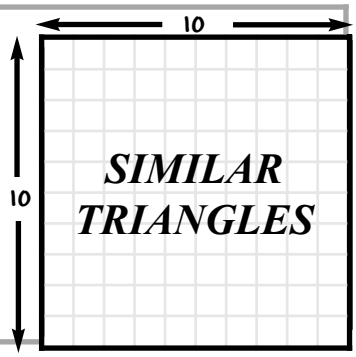
Lesson 6

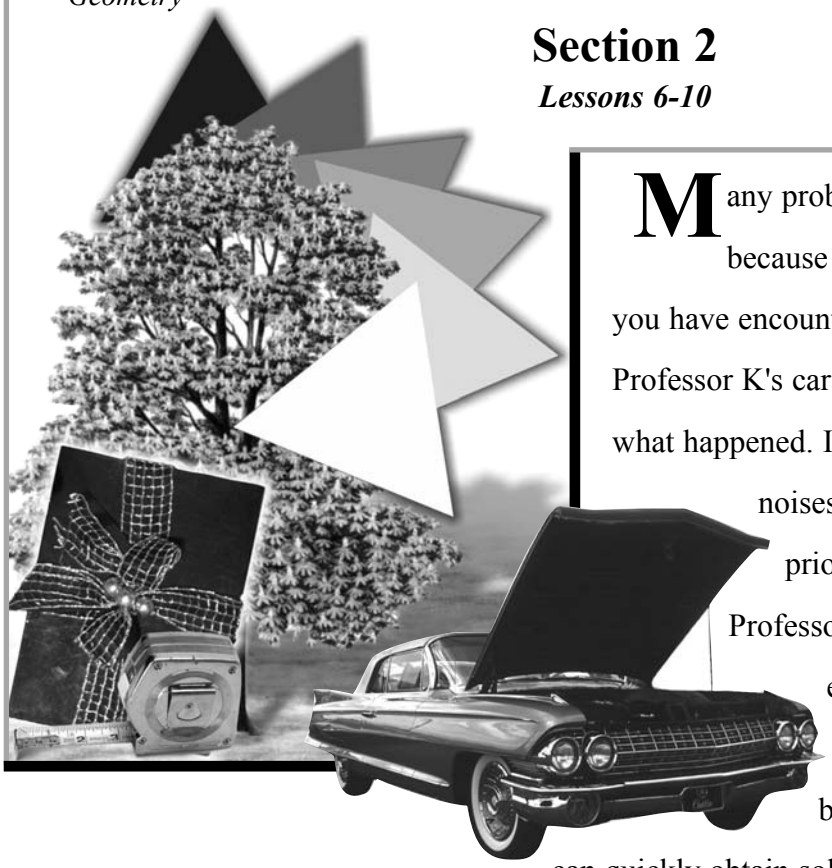
Geometry

Chapter 1

Section 2

Lessons 6-10





Many problems in life can be solved because they are similar to problems you have encountered in the past. When Professor K's car does not start, he thinks about what happened. If the engine made the same noises in this instance as possibly at prior times when the engine died, Professor K goes to that area of the engine and looks again to see if that is the problem. Often because we use similarities, we can quickly obtain solutions to the problem at hand.

In mathematics similarities are used to find solutions to problems. Professor K will describe ***similar*** triangles in this lesson and use them in the next lesson to determine solutions to problems.

All triangles are similar in that they have three sides and three angles. However, for two triangles to be similar mathematically speaking, the ***corresponding*** angles of the triangles must be ***congruent***. When two triangles are similar the ratios of the corresponding sides are also equal. This is shown in the definition of similar triangles on the next page.

VOCABULARY

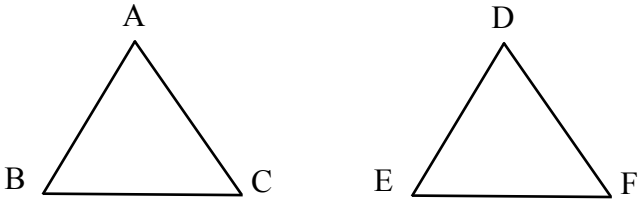
Similar: (~) having the same shape but not necessarily the same size
Corresponding: in the same respective location
Congruent: (≅) having the same measure, as in ∠A is congruent to ∠B because the measure of both angles is 45°
Reflexive property: property of mathematics which states, if $a = b$ and $b = c$, then $a = c$

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Definition: Congruent



Two triangles are similar if and only if each of the corresponding angles are congruent and the ratios of corresponding sides are equal. Consider $\triangle ABC$ and $\triangle DEF$ below.

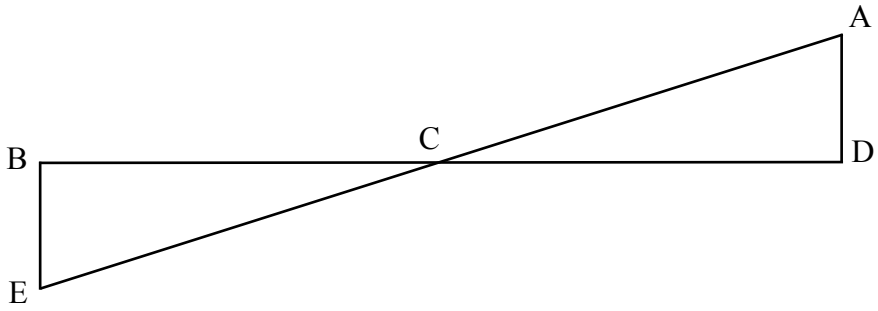


If $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$ then $\triangle ABC \sim \triangle DEF$.

If $\triangle ABC \sim \triangle DEF$, then $\frac{AB}{DE} = \frac{AC}{DE} = \frac{BC}{EF}$.

Look at **Example Set 1** as Professor K explains further the definition of similar triangles.

Example Set 1



Consider the two triangles formed by the intersecting lines AE and BD. Lines are drawn so that $\angle B$ and $\angle D$ are right angles (90°). Show that $\triangle ACD \sim \triangle ECB$.

Professor K remembers that intersecting lines form vertical angles. Professor K also remembers that vertical angles are congruent. Therefore, $\angle BCE \cong \angle ACD$.

Professor K uses the fact that the sum of the measures of the angles of a triangle equals 180°. This means that $m\angle EBC + m\angle BCE + m\angle CEB = 180°$ and $m\angle ADC + m\angle ACD + m\angle CAD = 180°$. Since the sum of the measures of the angles of the two triangles both equal 180°, Professor K uses the ***reflexive property*** (If a = c and b = c, then a = b) and writes the following equality.

$$m\angle EBC + m\angle BCE + m\angle CEB = m\angle ADC + m\angle ACD + m\angle CAD$$

Professor K can substitute 90° for the $m\angle EBC$ and $m\angle ADC$ by the definition of right angles. Also, since $\angle BCE \cong \angle ACD$, then $m\angle BCE = m\angle ACD$. Professor K can also substitute $m\angle ACD$ for the $m\angle BCE$ as shown.

$$90° + m\angle ACD + m\angle CEB = 90° + m\angle ACD + m\angle CAD$$

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Professor K begins to solve the equation by subtracting 90° and the m∠ACD from both sides.

$$\cancel{90°} - \cancel{90°} + \cancel{m\angle ACD} - \cancel{m\angle ACD} + m\,CEB = \cancel{90°} - \cancel{90°} + \cancel{m\angle ACD} - \cancel{m\angle ACD} + m\angle CAD;$$
$$m\angle CEB = m\angle CAD$$

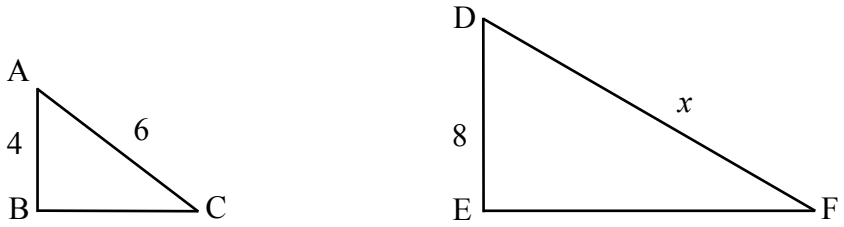
As shown after the subtractions, m∠CEB = m∠CAD; therefore, ∠CEB ≅ ∠CAD.

At this point Professor K has shown that ∠CEB ≅ ∠CAD, ∠BCE ≅ ∠ACD and ∠EBC ≅ ∠ADC. Therefore, Professor K can say that △ACD ~ △BCE, because corresponding angles are congruent in the two triangles.

Professor K showed you in **Example Set 1** how to show that two triangles are similar using a property known as Angle-Angle-Angle (A-A-A), or since three angles of a triangle are congruent then the two triangles are similar.

In **Example Set 2**, Professor K will show how the second portion of the similar triangle definition is used to find the length of an unknown side of one of the triangles.

Example Set 2



If △ABC ~ △DEF, find the length of the side DF.

Professor K knows from the definition of similar triangles that the ratios of corresponding sides of similar triangles are equal. Therefore, Professor K set up the following ratios using the variable x as the length of side DF.

$$\frac{AB}{DE} = \frac{AC}{DF} \rightarrow \frac{4}{8} = \frac{6}{x}$$

Professor K now solves this equation for x by first multiplying both sides by 8 and then by x, obtaining the following equation.

$$\frac{4 \bullet \cancel{8} \bullet x}{\cancel{8}} = \frac{6 \bullet 8 \bullet \cancel{x}}{\cancel{x}} \rightarrow 4x = 48$$

Dividing both sides by 4, Professor K finds that the length of the side DE is 12 units.

$$\frac{4x}{4} = \frac{48}{4} \rightarrow x = 12$$

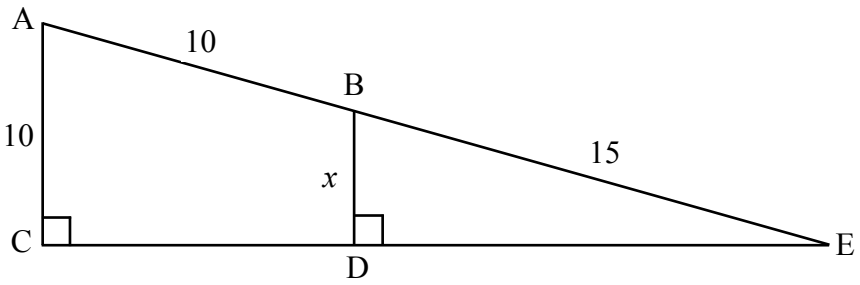
Professor K points out the another way to look at arriving at the equation $4x = 48$ is to "cross multiply", $\frac{4}{8} = \frac{6}{x}$.

In **Example Set 3** Professor K shows how to find the length of a missing side in similar triangles that are presented in a different form. Follow along as Professor K works through this example for you.



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Example Set 3



Notice that these two triangles are similar because both contain a right angle and both contain the ∠E. By using the same proof as in Example Set 1, Professor K knows that ∠CAE = ∠DBE. Therefore, ∠ACE ≅ ∠BDE, because of the A-A-A (three corresponding angles equal) property of similar triangles. Professor K then sets up the following ratios to determine the length of the line segment BD.

$$\frac{AC}{AE} = \frac{BD}{BE} \rightarrow \frac{10}{25} = \frac{x}{15}$$

Professor K wants you to notice that the length of the line segment, AE, was determined by adding the length of AB to BE (10 + 15 = 25). Professor K cross multiplies to obtain the equation: $25x = 15(10)$. Solving for x by dividing both sides of the equation by 25, Professor K determines that $x = 6$.

Show Professor K you understand how to determine the length of the sides of similar triangles by working the activities for this lesson.

LIFE PRINCIPLE



"Where there is matter, there is geometry."

—Johannes Kepler

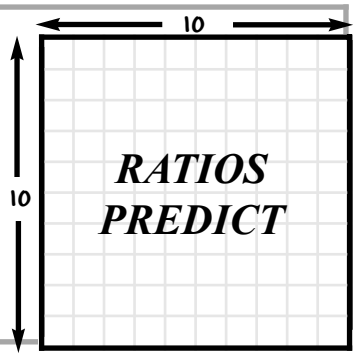
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Chapter 1



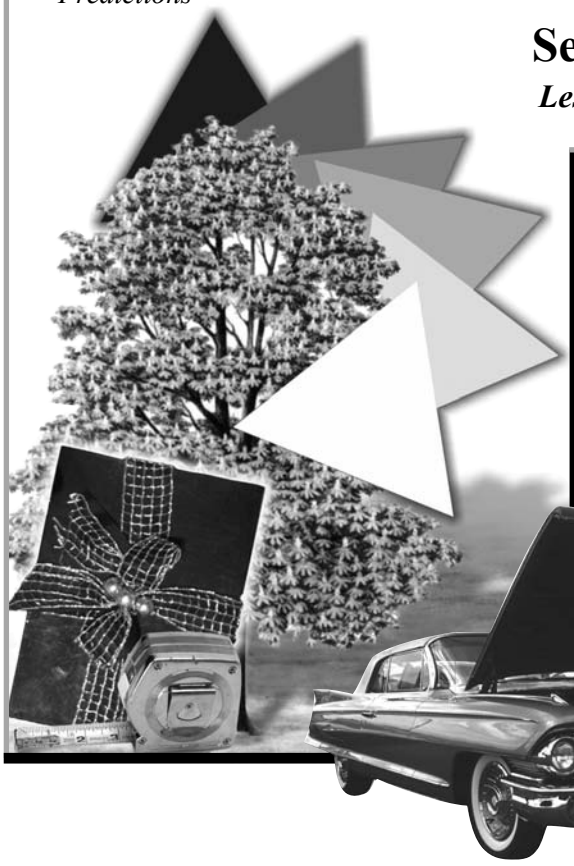
Lesson 7

Predictions



Section 2

Lessons 6-10



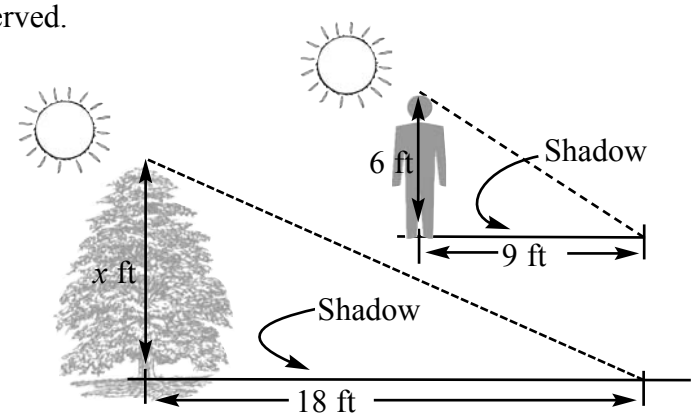
In certain areas of mathematics you can use similar objects to predict or determine the size of the object in question. Professor K will show you in this lesson how you can take what you have learned in the previous lesson and by use of ***similarities***, determine missing lengths in similar objects.

Michael, one of Professor K's neighbors, has a tree in his backyard. Michael asked Professor K if he could help him determine how tall the tree was. Professor K told his friend that he would be over during the day when there would be a good shadow cast by the tree onto the ground. Professor K and Michael got measuring tapes, paper and pencils and proceeded to determine the height of the tree.

Follow the steps in **Example Set 1** that Professor K used to estimate the height of the tree in Michael's backyard.

Example Set 1

Professor K and Michael went to Michael's backyard and proceeded to make the following sketch of what they observed.



VOCABULARY

Similarities: like characteristics but not the same
Proportion: a ratio; fraction of a whole
Reductions: the results of reducing a fraction to lowest terms

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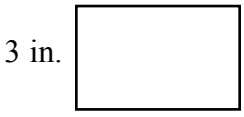
Professor K stood in the sunlight and Michael measured Professor K's shadow to be nine feet in length. Michael then measured Professor K's height and found that Professor K was six feet tall. Michael and Professor K then measured the length of the shadow cast by the tree and determined it to be 18 feet in length. Professor K knew that a ***proportion*** could be set up to determine the height of the tree based on the information that he and Michael had obtained. This is possible because as you can see the triangles formed are similar or look the same. Professor K set up a ratio of the lengths of corresponding sides as shown below.

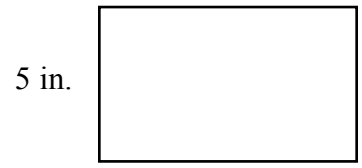
$$^{x}/_{18} = {}^{6}/_{9} \rightarrow 18(6) = 9x \rightarrow {}^{18(6)}/_{9} = {}^{9x}/_{9} \rightarrow 2(6) = x$$

Through cross-multiplying and division, Professor K determined that the height of his neighbor's tree was 12 feet.

Example Set 2

Another important use of similarities is the enlargement of photographs or copies on a copy machine. These objects can be enlarged or reduced by use of ratios because the shapes are similar. Usually these shapes are squares or rectangles. Consider the photo paper below and its enlargement. Determine the length of the missing side.







Professor K sets up a ratio making sure that corresponding dimensions are placed in the proper place in the fractions.

$$^{3}/_{5} = {}^{5}/_{x} \rightarrow 5 \bullet 5 = 3x \rightarrow {}^{25}/_{3} = {}^{3x}/_{3} \rightarrow x = 8\,{}^{1}/_{3}$$

After cross multiplication, division by 3 and ***reduction*** of the fraction, Professor K determines that proportionally, the length of the missing side should be $8\,{}^{1}/_{3}$ inches.

Show Professor K you can apply ratios to similar shapes in real life problems by working the activities for this lesson.

Life Principle



"The height of a man's wisdom is determined by his character."

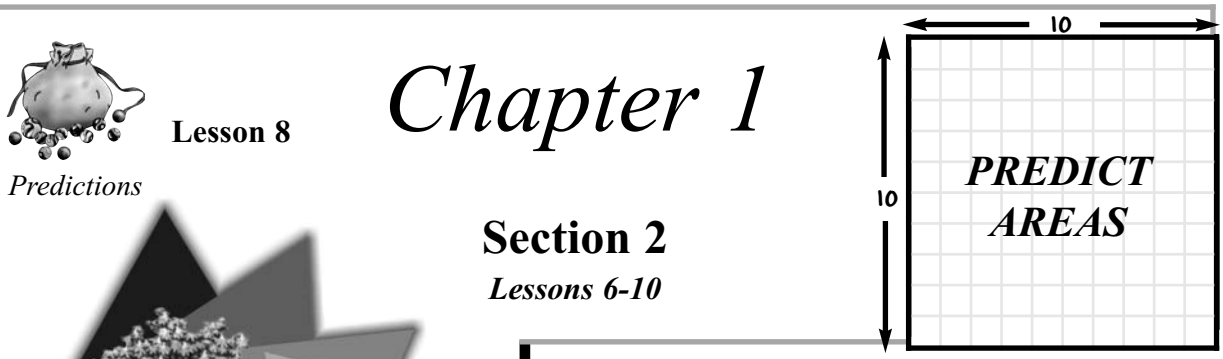
—Anonymous

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Chapter 1

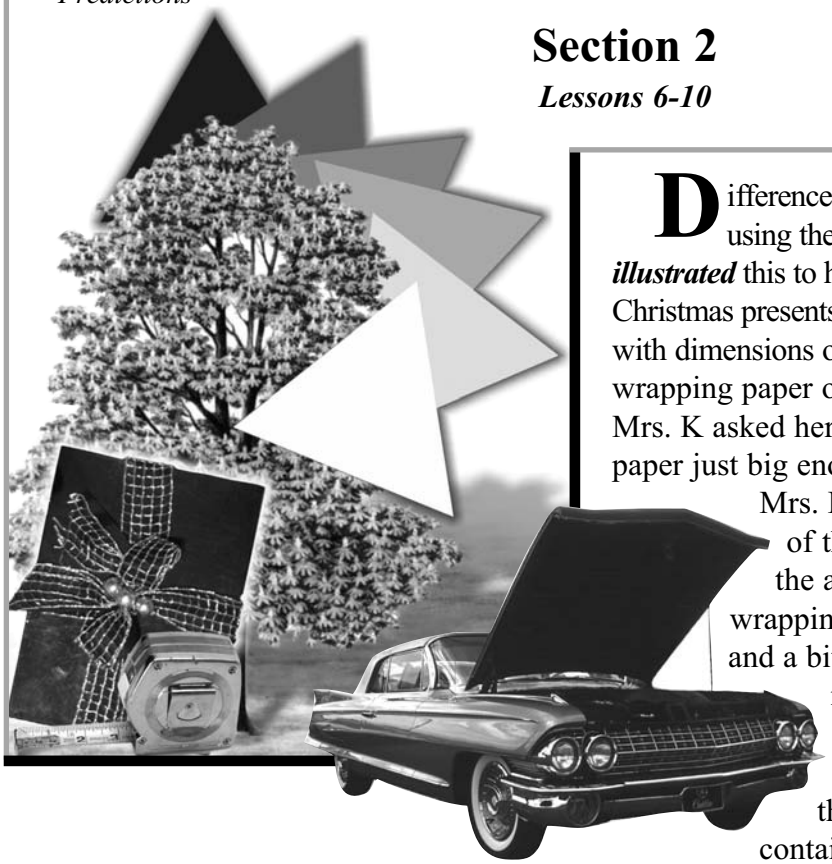
Lesson 8

Predictions



Section 2

Lessons 6-10



Differences in area can be calculated by using the concept of similarity. Professor K ***illustrated*** this to his wife while she was wrapping Christmas presents. Mrs. K had just wrapped a box with dimensions of 4 in. × 4 in. × 6 in., using wrapping paper of dimensions 10 in. × 20 in. Mrs. K asked her husband to get some wrapping paper just big enough to cover another box.

Mrs. K told Professor K that the area of the paper needed to be one-half the area of the previous sheet of wrapping paper. Mrs. K was surprised, and a bit upset, when her husband returned quickly with several sheets of wrapping paper. Professor K explained that all the sheets of wrapping paper contained the required area, but the ***dimensions*** were different. Also, since Mrs. K had not specified any particular width or length, he found several sheets of wrapping paper that contained the proper area. Professor K told his wife that the calculations were easy when you understand the concept of similar objects.

Look at **Example Set 1** as Professor K shows you how he obtained the answers quickly to the problem that his wife ***posed*** to him. Be sure to follow the steps closely so that you too can solve similar types of problems.

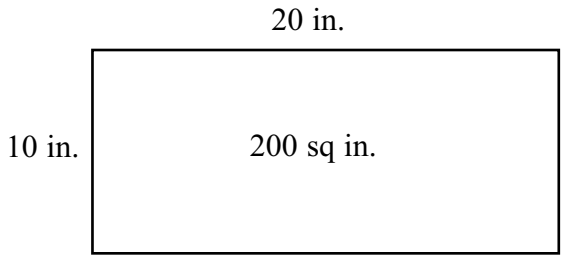
VOCABULARY

Illustrate: to demonstrate in some way
Dimension: a measure in one direction
Posed: presented to

Example Set 1

Professor K knew that the first sheet of wrapping paper was 20 in. × 10 in., and therefore, the area was 200 sq in. (10 × 20 = 200).

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Professor K calculated the area of the sheet of paper Mrs. K desired to be 100 sq in. ($\frac{1}{2} \bullet 200 = 100$). Professor K then began a series of quick calculations using different lengths for width and height to obtain an area of 100 sq in. Below is a table containing Professor K's calculations. Remember, $A = l \times w$

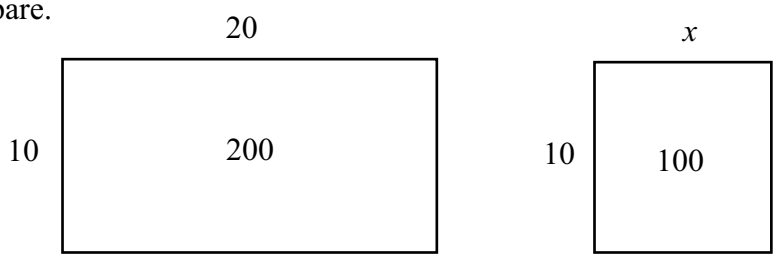
Width	Length	Area
1	100	$1 \times 100 = 100$
2	50	$2 \times 50 = 100$
3	$33\frac{1}{3}$	$3 \times 33\frac{1}{3} = 100$
4	25	$4 \times 25 = 100$
5	20	$5 \times 20 = 100$
8	$12\frac{1}{2}$	$8 \times 12\frac{1}{2} = 100$
10	10	$10 \times 10 = 100$

Professor K performed his calculations as shown and proceeded to discard the wrapping paper with widths 1, 2, and 3 inches because it was unreasonable to assume that his wife would need a sheet of paper that narrow and that long to wrap the gift. Professor K therefore, brought his wife the four sheets with widths of 4, 5, 8 and 10 inches.

Mrs. K told Professor K that he had brought several sheets of paper not knowing which one would actually fit the dimensions of the box. Mrs. K told her husband that it appeared to her that it would have been easier to have just used ratios. Mrs. K showed her husband how she would have used ratios to arrive at the answer even more quickly. Look at **Example Set 2** to see how Mrs. K demonstrated this to Professor K.

Example Set 2

Mrs. K determined that the sheets of wrapping paper should have similar shapes, rectangles. She set up a ratio to determine the length of one of the sides compared to the corresponding side of the other rectangle and the ratio of the areas. Mrs. K set up the proportion for solving the problem shown below. Mrs. K chose the length as the side of the rectangle to compare.



$\frac{20}{200} = \frac{x}{100} \rightarrow 20 \bullet 100 = 200x \rightarrow \frac{2000}{200} = \frac{200x}{200} \rightarrow x = 10$

After cross-multiplying, division by 200 and reducing the fraction, Mrs. K determined that proportionally the length of the missing side should be 10 inches.

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Show Professor K you can apply ratios to the areas of similar shapes in real life problems by working the activities for this lesson.

Life Principle



"Man is like a fraction whose numerator is what he is and whose denominator is what he thinks of himself. The larger the denominator, the smaller the fraction."

—Count Leo Tolstoy

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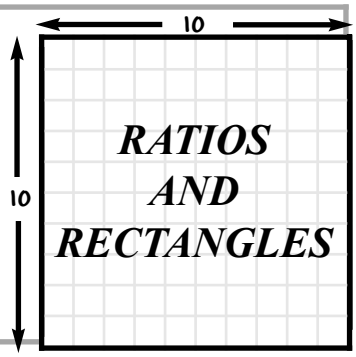
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Lesson 9

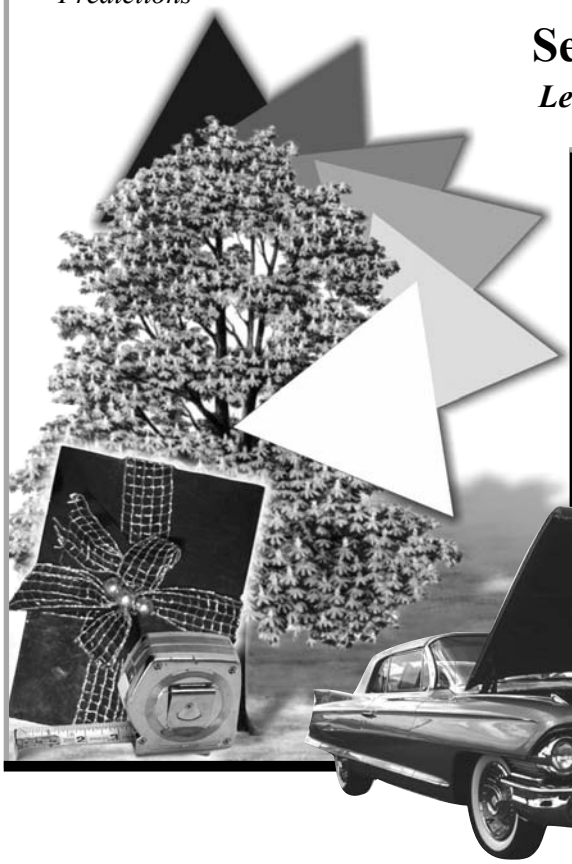
Predictions

Chapter 1



Section 2

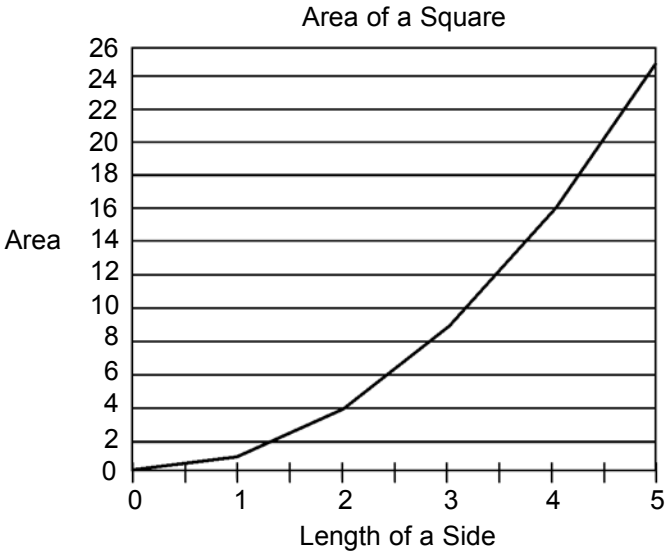
Lessons 6-10



In the previous lesson, Professor K demonstrated that if you change just one side of a rectangle then the area is affected by the exact same factor and vice versa. Professor K will show you in this lesson what happens when both sides of a rectangle are reduced or enlarged by the same ***factor***. Look at the graph that Professor K has drawn for you below so that you can visualize how changing both sides of a rectangle effects the area.



Professor K



VOCABULARY

Factor: one of the numbers that are multiplied together in a multiplication problem

Double, doubling: to make twice as large; two times

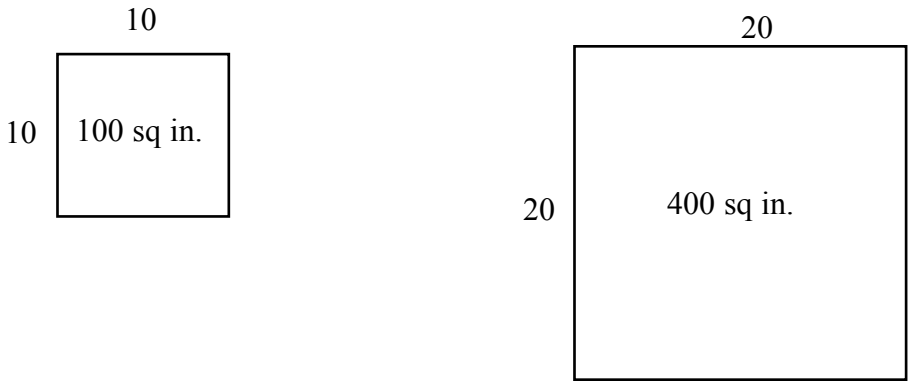
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Notice that as the length of the side of the square changes by only one, the area goes up more quickly. The graph is not that of a straight line, but instead a curve. The area is changing by the power or exponent of two. Refer to the graph as Professor K explains this.

If a square has a side of length one, then the area is $1 \times 1 = 1^2 = 1$. If a square has a side of length 2, then the area of the square is $2 \times 2 = 2^2 = 4$. Notice the length of the side was doubled but the area increased by four times. Consider a square with a side of length 4. This is ***double*** the square of side length 2. Looking at the graph you can see that the area of a square with a side of length 4 is 16. Again, the length of a side was doubled, but the area increased by a factor of four.

Look at **Example Set 1** as Professor K shows you the relationships of sides and areas. Also you will see how and when doubling occurs in area.

Example Set 1

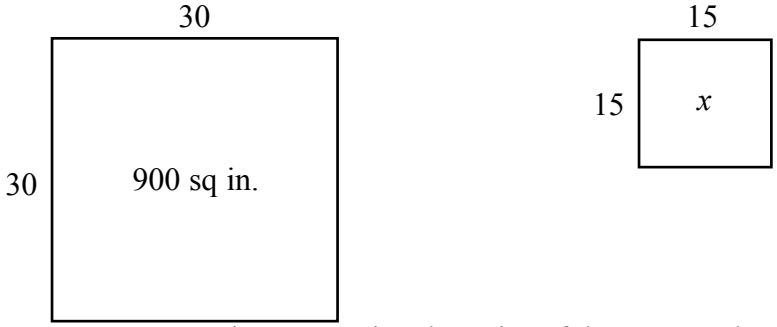


Professor K shows you that the first square is 10 inches on each side. The area of that square is 100 square inches (10 in. × 10 in. = 100 sq in.). The second square is 20 inches on each side (twice the length of the first square) and the area is 400 square inches (20 in. × 20 in. = 400 sq in.) or four times larger than the area of the first square.

Look at **Example Sets 2 and 3** to see how Professor K calculates areas quickly using the fact that areas change by factors containing an exponent of 2. Follow Professor K's steps closely so that you can solve such problems.

Example Set 2

Mrs. K wanted a sheet of wrapping paper exactly one-half the length on a side as the one shown below. Calculate the area of the second sheet of wrapping paper using ratios.



Professor K sets up a proportion comparing the ratios of the areas to the ratios of the sides.

$\frac{x}{900} = (\frac{15}{30})^2 \rightarrow \frac{x}{900} = (\frac{1}{2})^2 \rightarrow \frac{x}{900} = \frac{1}{4} \rightarrow 4x = 1 \bullet 900 \rightarrow \frac{4x}{4} = \frac{900}{4} \rightarrow x = 225$

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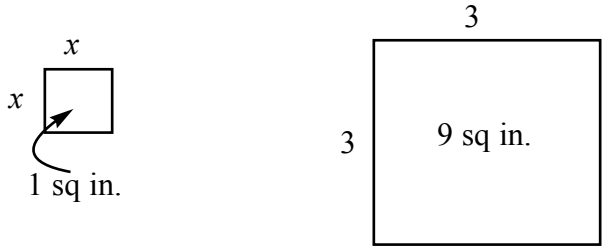
Professor K uses PEMDAS to begin this problem. Professor K reduces the fraction within the parentheses (P) first. Then Professor K squares the ratio of the lengths (exponents, E). Then after cross-multiplying, division by 4 and reducing the fraction, Professor K determined that proportionally, the area of the second square should be 225 square inches.

Notice in **Example Set 2** if Professor K had not squared the ratio of the lengths, he would have obtained 450 sq in. as the answer. Since the second square contained sides $\frac{1}{2}$ the length of the first, the length of the sides of the second square was 15 inches. To check Professor K's answer in **Example Set 2** you can calculate the area of a square of each side measuring 15 inches. That area is: 15 in. × 15 in. = 225 sq in.

Professor K reminds you that you must square the ratio that determines the enlargement or reduction in an area problem when the length of both sides change. If the sides change by a factor of three then the areas change by a factor of nine (3^2). If the length of a side changes by a factor of 4, then the area changes by a factor of 16 (4^2) and so forth.

Example Set 3

A square is one-ninth the area of a second square. Calculate the length of the side of the first square.



Professor K sets up a proportion comparing the ratios of the areas to the ratios of the sides.

$$\frac{9}{1} = \left(\frac{3}{x}\right)^2 \rightarrow \frac{9}{1} = \frac{9}{x^2} \rightarrow 9x^2 = 9 \bullet 1 \rightarrow \frac{9x^2}{9} = \frac{9}{9} \rightarrow x^2 = 1 \rightarrow x = 1$$

Professor K uses PEMDAS to begin this problem. Professor K determines that no reductions are possible of the fraction within the parentheses (P) first. Then Professor K squares the ratio of the lengths (exponents, E). Then after cross-multiplying, division by 9 and reducing the fraction, Professor K determined that $x^2 = 1$. Professor K then took the square root of both sides. (Remember what you do to one side, you must do to the other.) The square root of 1 = 1, therefore, $x = 1$. Professor K determined that proportionally, the length of a side of the first square should be 1 inch.

Show Professor K you can apply ratios to the areas of similar shapes in real life problems by working the activities for this lesson.

LIFE PRINCIPLE



"Small problems become large, when not dealt with quickly."

—Anonymous

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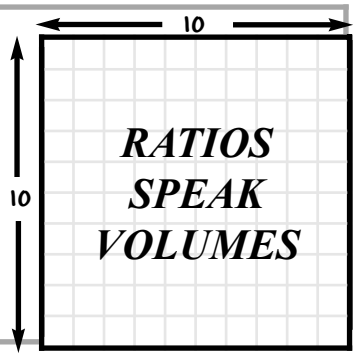
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Lesson 10

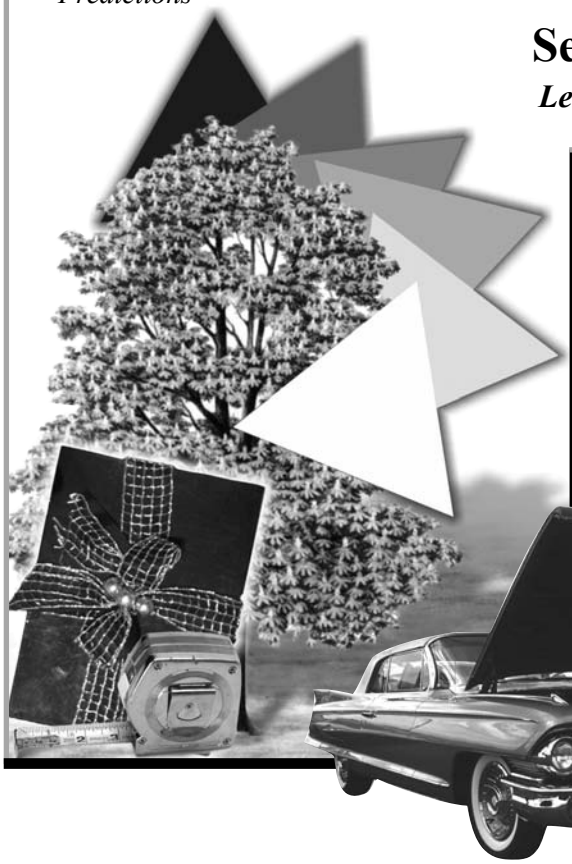
Predictions

Chapter 1



Section 2

Lessons 6-10



Differences in volume can be calculated by using the concept of *similarity*, just as area was. Professor K's wife wanted Professor K to find a box one-half the volume of the 4 in. × 4 in. × 6 in. box that was in the previous lesson. This task required a little more knowledge of similar shapes than did calculating areas. Professor K remembered that volume is in ***cubic*** units. That means that if the object happens to be a cube, you simply cube one edge and you have the volume of the object. Look at **Example Set 1** as Professor K refreshes your memory to this concept.



Professor K

VOCABULARY

Cubic: referring to the shape of a cube
Factor of reduction: the factor used to reduce the size of something
Triple: times three
Factor of enlargement: the factor used to enlarge the size of something

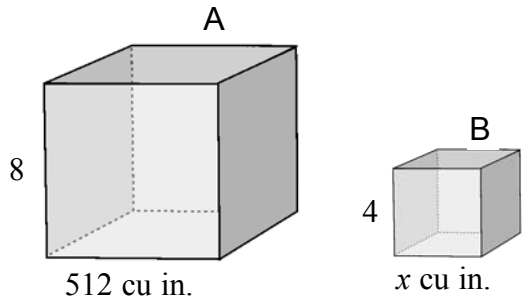
Example Set 1

A cube has dimensions of 2 in. × 2 in. × 2 in. Calculate the volume of the cube. $V = l \times w \times h$ = 2 in. × 2 in. × 2 in. = $(2 \text{ in.})^3$ = 8 cubic in. or 8 in.^3

Professor K knows that it is important for you to realize that the units and values of volumes are cubic in nature. Therefore, when making calculations of the volume of similar shapes, the desired ratio is cubed or raised to the power of three. Follow the steps closely as Professor K demonstrates this by working the problem in **Example Set 2**.

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Example Set 2



The dimensions of cube A are reduced by one-half for cube B. What is the volume of cube B? Professor K has included the length of an edge of each cube as a way to check and show that the procedure that he shows in this example does indeed obtain the correct answer. Since volumes are cubic, the ***factor of reduction***, $^1/_2$ must be cubed. Therefore, the new factor of reduction is $(^1/_2)^3 = {}^{(1)^3}/_{(2)^3} = {}^{1 \bullet 1 \bullet 1}/_{2 \bullet 2 \bullet 2} = {}^1/_8$. Professor K takes the new reduction factor and multiplies it times the original volume, 512 cubic inches.

$$512 \text{ cu in.} \times {}^1/_8 = {}^{512}/_1 \times {}^1/_8 = {}^{512}/_8 = 64 \text{ cubic inches}$$

Professor K performs the calculations as shown to obtain 64 cubic inches as the volume of the cube that is $^1/_2$ the dimensions of the first cube.
Professor K checks his answer by using the dimension given for cube B.

$$4 \text{ in.} \times 4 \text{ in.} \times 4 \text{ in.} = 64 \text{ cu in.}$$

Cubing the length of the edge of cube B obtains the same result as the reduction based on similar objects.

In **Example Set 3**, Professor K shows that the opposite of reduction (enlargement) can also be accomplished using ratios of similar objects. Watch closely as Professor K demonstrates this concept.

Example Set 3

A cube has a volume of 9 cubic meters. If the length of an edge is ***tripled***, determine the volume of the new cube. Professor K uses the same principle of cubes for enlargement as he did for reduction. The ***factor of enlargement*** must be cubed. Therefore, since the larger cube has dimensions three times the smaller, the volume increases by twenty-seven times ($3^3 = 27$). Professor K uses this factor to work the problem.
9 cu meters × 27 = 243 cubic meters.

Show Professor K you can apply ratios to the volumes of similar shapes in real life problems by working the activities for this lesson.

LIFE PRINCIPLE



"Reducing a large problem into smaller problems, enlarges the probability of success."

—Anonymous

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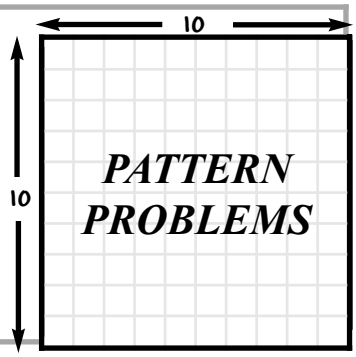
Lesson 11

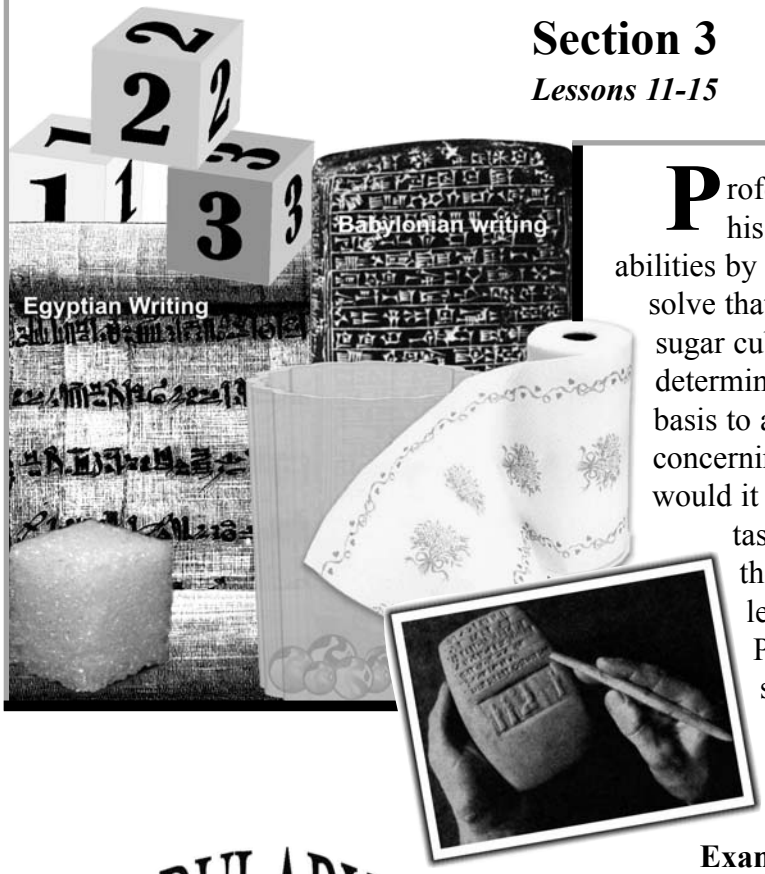
Predictions

Chapter 1

Section 3

Lessons 11-15







Professor K enjoyed testing his children's thinking abilities by giving them problems to solve that used objects such as sugar cubes as a basis for determining a ***pattern***. Professor K used this basis to ask questions of his children concerning how many more sugar cubes would it take to accomplish the different tasks that Professor K would present to them. Look at the examples in this lesson to see some of the problems Professor K asked his children to solve. Then look at the explanation of how the correct answer is determined by thinking through the problem.

VOCABULARY

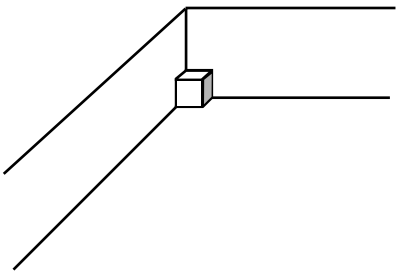
Pattern: arrangement of design, predictable sequence
Exposed faces: the faces of a solid shape that are not covered, or visible
Initial: the way a thing began
Layer: the number of items necessary to cover all the exposed surfaces of a 3-dimensional object (such as a cube), or sides only of a 2-dimensional object (such as a triangle)

Example Set 1

Professor K placed one sugar cube in the corner of a shoebox. He then asked his children to determine how many cubes that it would take to cover all of the ***exposed faces*** of the first cube.

Step 1

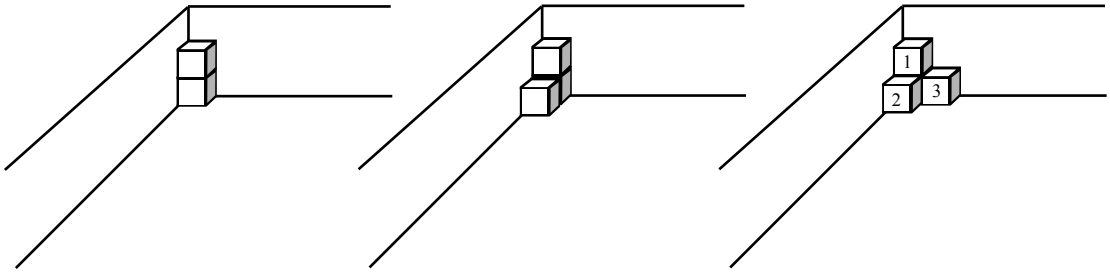
Professor K's children visualized what the ***initial*** problem looked like.



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Step 2

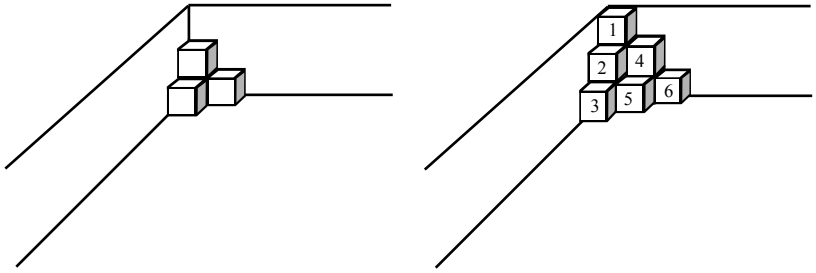
The children then noticed that when looking at the sugar cube in the corner of the shoebox they could see only three faces. These were the exposed faces. Since three of the six faces of the sugar cube were exposed, it took three sugar cubes to cover them. See the diagram below.



Professor K explained to his children that their answer of three sugar cubes to cover all the exposed faces was correct. He explained further that the number of sugar cubes that it took to cover all the exposed faces is known as a ***layer***.

Professor K's next question was to determine how many cubes would be in the second layer. Professor K's children used the diagram from **Example Set 1** as their starting point and visualized the next layer of sugar cubes. See how they did this and determined the answer in **Example Set 2**.

Example Set 2

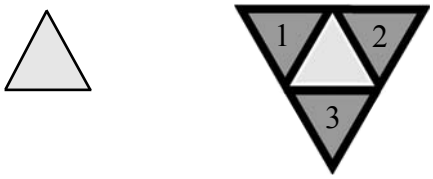


Professor K's children started with four cubes in the corner of the box. They then proceeded to cover all the exposed faces with a cube and determined that it would take six sugar cubes for the second layer of Professor K's puzzle problem.

In **Example Set 3** Professor K shows how his children answered another shapes-and pattern-problem that he offered them.

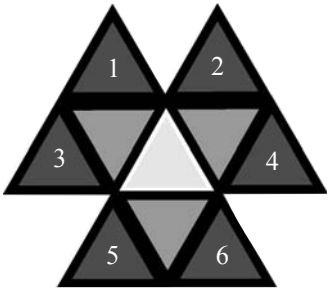
Example Set 3

A triangle is drawn on a piece of paper. Determine how many triangles, congruent to the first, are in the second layer. In a flat, 2-dimensional object such as a triangle, a layer consists of triangles drawn attached to the exposed sides (not tops) of the outer triangles. Look below to understand what Professor K's children drew to answer this problem.



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Professor K's children placed three triangles as shown to form the first layer. They then formed the second layer of triangles in the same fashion as shown below.



The four shaded triangles in the middle are the triangles from the initial layer and layer one. The outer black triangles are triangles required for the second layer. Therefore, the answer to Professor K's question is six triangles make up the second layer.

Show Professor K you can work pattern problems with shapes by working the activities for this lesson.

LIFE PRINCIPLE



"All of a man's deeds, good or bad, will come to light."

—Anonymous

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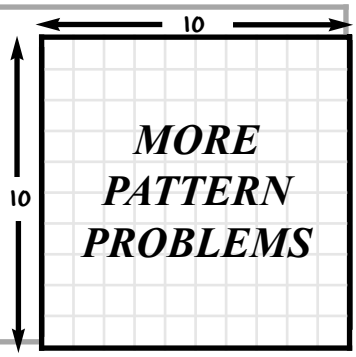
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Lesson 12

Predictions

Chapter 1



Section 3

Lessons 11-15



Professor K wanted his children to attempt to develop a mathematic model that would calculate the number of sugar cubes in each new layer. Professor K had confidence in his children's thinking abilities and encouraged them to give their best effort. Professor K used the same problems that he used in the previous lesson. However, this time Professor K asked his children to determine the number of cubes in a greater level using a model or formula that they had determined from actually counting the cubes in previous layers. This process is looking at what you know and make a ***conjecture*** as to what the solution will be. After making the conjecture, you must check it against something for which you can obtain the answer another way, such as actually counting the sugar cubes of a layer. Look at the examples in this lesson to see some of the problems Professor K asked his children to solve. Be sure to look at the explanation of how the correct answer is determined by thinking through the problem and then checking it.



VOCABULARY

Conjecture: an unconfirmed idea, a guess
Trial-and-error: trying different things until something succeeds
Derived: deduced, thought of

Table 12.1

# Layers	# cubes per layer	# total cubes
1	1	1
2	3	4
3	6	10
4	10	20

Table 12.1 is a record of the results from the problem involving the sugar cubes from the previous lesson. By making a list of the numbers it is easier to see the pattern that

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the problem is following to obtain the number of cubes in each layer. Professor K's children looked at the list of numbers in the second column (1, 3, 6, 10), then attempted to determine a formula that would obtain the next number in the list. One observation is that you can add 2 to the first number to obtain the second; add 3 to the second number to obtain the third; add 4 to the third number to obtain the fourth, and so on. This is true, but the challenge is to put that reasoning into a workable formula. To begin this task, Professor K had taught his children to use a variable to represent the number of the term. The letter, n, is usually the letter that is used to represent the number of the term. A workable formula is usually obtained by using a ***trial-and-error*** system. The value of the letter n must be specified. Look at **Example Set 1** to see the trial-and-error process that Professor K's children used to obtain a formula for the number of sugar cubes in the next layer.

Example Set 1

Determine a formula for calculating the next term in the list of numbers, 1, 3, 6, 10.

Step 1

Professor K's children assigned n the values 0, 1, 2, 3, 4, 5, …. They then calculated the results using a formula they determined might work the problem. $2n + 1$.

$$n = 0;\ 2n + 1 = 2(0) + 1 = 0 + 1 = 1$$



This value is the same as the first term. Good so far.

$$n = 1;\ 2n + 1 = 2(1) + 1 = 2 + 1 = 3$$

This value is the same as the second term. Good so far.

$$n = 2;\ 2n + 1 = 2(2) + 1 = 4 + 1 = 5$$

This value is not the same as the third term. Start over.

Step 2

The children knew that they had to start over because their proposed formula failed to check for the third term. The children ***derived*** another formula. They thought the formula, $n(n + 1)$ where $n = 0, 1, 2, 3, 4$ might work.

$$n = 0;\ n(n + 1) = 0(0 + 1) = 0$$

The test failed quickly.
The children changed the values of n to 1, 2, 3, 4, 5, …. These values are the best to use for n because the numbers correspond to the number of the term in question.

$$n = 1;\ n(n + 1) = 1(1 + 1) = 1(2) = 2$$

This test also failed, but Professor K's children noticed something very interesting. The value of the first term is one-half that of the answer obtained from the formula. Therefore, Professor K's children revised their last formula to read: $^{n(n+1)}/_{2}$. This formula divides the previous formula by two. The children now check their new formula to see if it will work using $n = 1, 2, 3, 4, 5$ ….

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$n = 1$; $n(n+1)/2 = 1(1+1)/2 = (1)2/2 = 2/2 = 1$

This value is the same as the first term.

$n = 2$; $n(n+1)/2 = 2(2+1)/2 = 2(3)/2 = 6/2 = 3$

This value is the same as the second term.

$n = 3$; $n(n+1)/2 = 3(3+1)/2 = 3(4)/2 = 12/2 = 6$

This value is the same as the third term.

$n = 4$; $n(n+1)/2 = 4(4+1)/2 = 4(5)/2 = 20/2 = 10$

This value is the same as the fourth term.

Professor K's children determined that the formula they chose returned the same result as the list of numbers. Therefore, they are confident that they have the correct formula. Professor K's children can now calculate any value of n.

$n = 100$; $n(n+1)/2 = 100(100+1)/2 = 100(101)/2 = 10100/2 = 5{,}050$

Therefore, 5,050 is the hundredth term.

Professor K's next question was to determine "How many cylinders would be in each layer if a layer consisted of the number of cylinders required to cover the faces (ends) of a cylinder?" See how Professor K's children accomplished this and determined the answer in **Example Set 2**.

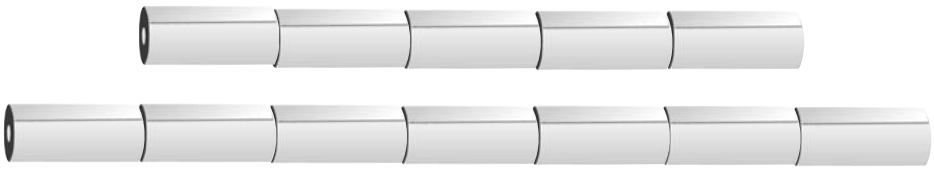
Example Set 2

Professor K laid a paper towel roll on the table. He asked his children to determine how many rolls would be in the twentieth layer, if a layer consisted of the number of rolls required to cover the faces of the cylinder.

Step 1



Professor K's children started with one paper towel roll and covered the ends (faces) with two paper towel rolls. They continued to do the same several times to obtain a number of outcomes.



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Step 2

Professor K's children made a list of their results using the following notation that Professor K had shown them one day so they could write their list in a shorthand way: (layer #, outcomes)

(1, 2), (2, 2), (3, 2), (4, 2)

Professor K's children realized that the number of rolls in all the layers after the initial layer would contain two rolls. They told Professor K that the formula would be $n(0) + 2$.

Step 3

Professor K thought that was too easy so he asked them to see if they could calculate a formula for the total number of rolls altogether after each layer. Professor K's children made another list of results: (layer #, total number of rolls)

(1,3), (2, 5), (3, 7), (4, 9)

Professor K's children determined the formula for the total number of rolls after each layer.

$2n + 1$

Show Professor K you can work pattern problems with shapes by working the activities for this lesson.

Life Principle



"We sometimes have trials in our lives because we fail to learn from the errors of the past."

—Anonymous

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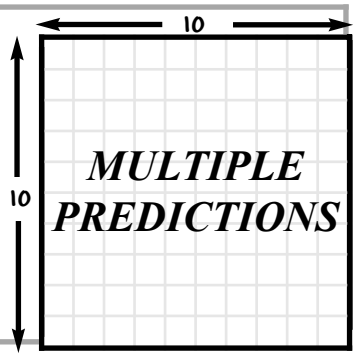
Lesson 13

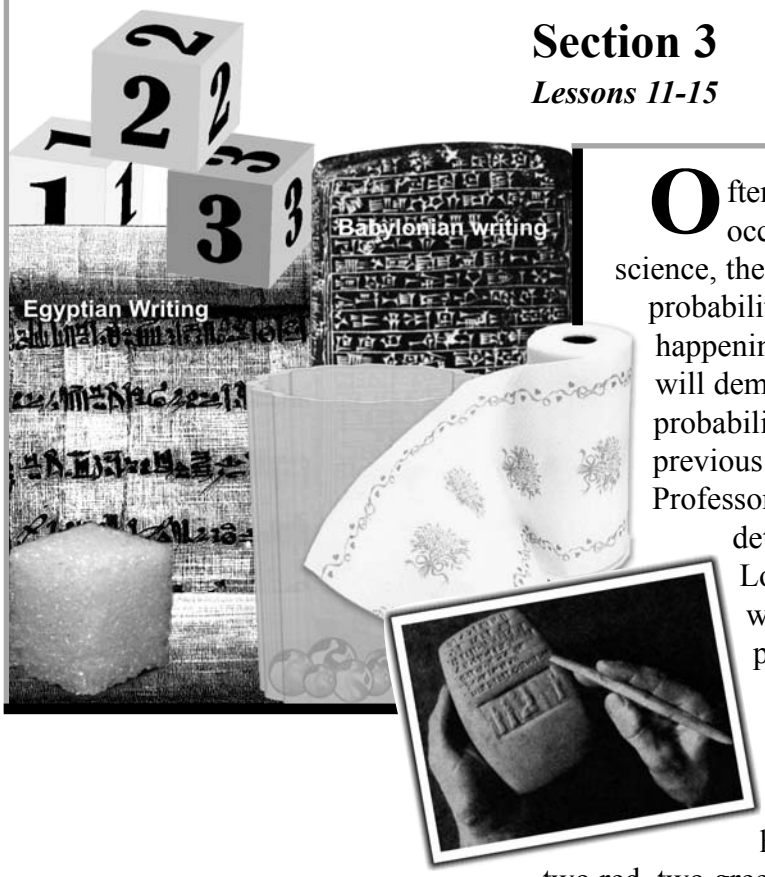
Predictions

Chapter 1

Section 3

Lessons 11-15





Often the probability of an outcome occurring more than once is desired. In science, the researcher wants to know the probability of an outcome of an experiment happening again. In this lesson, Professor K will demonstrate how to calculate the probability of an outcome occurring after previous outcomes. Follow along with Professor K as he explains the process of determining ***multiple*** probabilities. Look at **Example Set 1** as Professor K works the first multiple probability problem.

Example Set 1

Professor K told his youngest son, Matthew, to get some marbles from his toy box. Matthew returned with two red, two green, two black, one white and one blue marble. Professor K placed the marbles in a cup. He then asked Matthew to take one marble from the cup. Calculate the probability that the first marble pulled from the cup was red.

Step 1

The sample space contains eight elements. The number of red marbles is two. The probability of picking a red marble was $2 \div 8$ or $\frac{2}{8}$. This ratio reduces to a *one out of four* chance that the first marble that Matthew picked would be red. Professor K placed the marble back in the cup. Calculate the probability that Matthew pulls a red marble from the cup.

VOCABULARY

Multiple: many or many times

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Step 2

The probability will be the same as Step 1, because Professor K returned the marble to the cup, making a sample space of eight elements and the number of favorable outcomes two. So as in Step 1, the ratio is $^2/_8$, or there is a *one out of four* chance that Matthew will pull a red marble from the cup.
Matthew did pull a red marble from the cup a second time. Calculate the probability that Matthew would pull two red marbles from the cup in two attempts.

Step 3

To calculate this probability, you must combine by multiplication the probability of the two outcomes mentioned. Therefore, the calculation would be:

$$^2/_8 \bullet {}^2/_8 = {}^1/_4 \bullet {}^1/_4 = {}^1/_{16}$$

The probability of Matthew pulling two red marbles from the cup in two attempts is *one out of sixteen*.



Professor K explained to his children that if Matthew were to continue to repeat the experiment, the total probability would be calculated by continuing to multiply by the ratio of the probability of the next outcome. In **Example Set 1** the marble was placed back into the cup. In **Example Set 2** Professor K shows you how to calculate probabilities if the marble is not returned to the cup as well as calculating multiple probabilities.

Example Set 2

Matthew pulled a green marble from the cup that contained 2 red, 2 green, 2 black, one white and one blue marble. He placed the green marble back in the cup and proceeded to pull a blue marble from the cup. Matthew did not return the blue marble to the cup. Matthew then pulled a black marble from the cup. Determine the probability of Matthew pulling a green marble, a blue marble and a black marble from the cup.

Step 1

Calculate the probability of each outcome separately. Remember there are eight elements in the sample space.
P(green) = $^2/_8 = {}^1/_4$
P(blue) = $^1/_8$
P(black) = $^2/_7$ (There are only 7 elements now because the blue marble was not returned to the cup)

Step 2

Calculate the probability, P(green, blue, black).
P(green, blue, black) = $^1/_4 \bullet {}^1/_8 \bullet {}^2/_7 = {}^2/_{224} = {}^1/_{112}$
Professor K multiplies the individual probability for each color marble together to obtain the probability that Matthew pulls a green, blue, and black marble from the cup under the conditions given.

Show Professor K you can work problems involving probability by working the activities for this lesson.

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Life Principle



"Proper preparation multiplies
your chances of success."

—Anonymous

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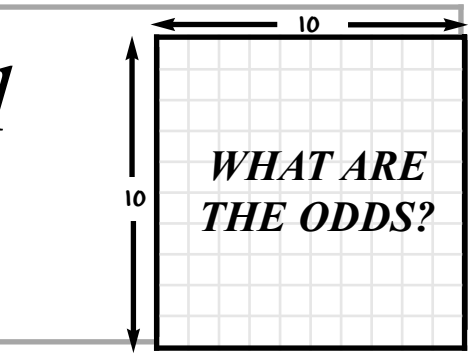
Chapter 1

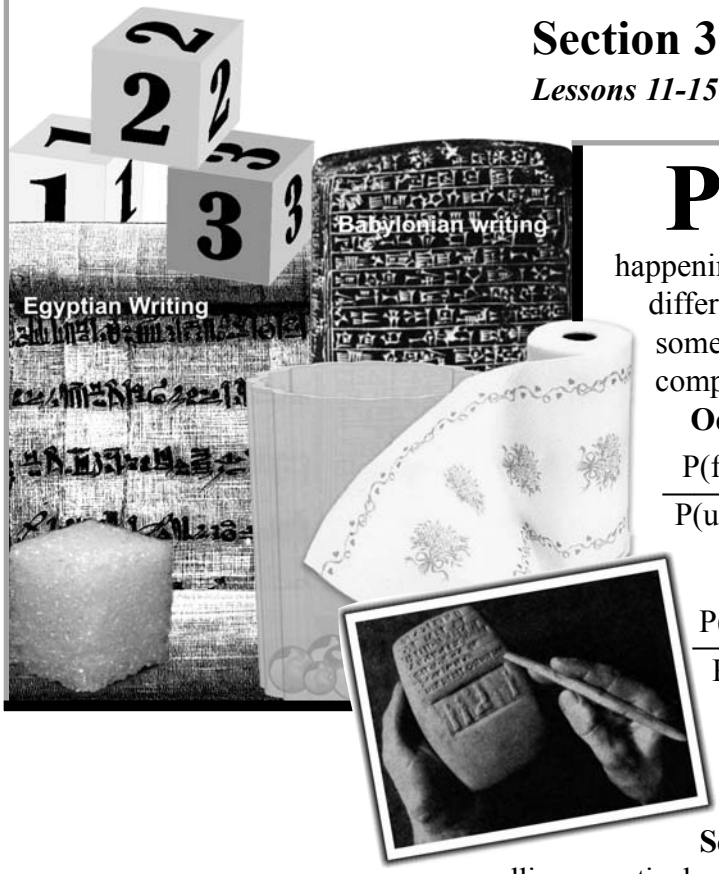
Lesson 14

Predictions

Section 3

Lessons 11-15





Professor K explained to his children that computing the odds of something happening or not happening is calculated differently than simply the chance of something happening. The formulas for computing odds are given below.

Odds in favor of an event occurring =

$$\frac{\text{P(favorable)}}{\text{P(unfavorable)}} = \frac{\text{\# of favorable outcomes}}{\text{\# of unfavorable outcomes}}$$

Odds against an event occurring =

$$\frac{\text{P(unfavorable)}}{\text{P(favorable)}} = \frac{\text{\# of unfavorable outcomes}}{\text{\# of favorable outcomes}}$$

Professor K will use the formulas to show you how to calculate *odds* and compare that ratio to probability. Look at **Example Set 1** as Professor K calculates the odds of pulling a particular number cube from a bag of 100 cubes.

Example Set 1

Determine the odds of pulling an even numbered cube from a bag containing 100 cubes labeled with only one number from 1 to 100.

Step 1 Professor K first has to calculate the *probability* of pulling an even numbered cube from the bag.

$$\text{P(even)} = \frac{50}{100} = \frac{1}{2}$$

Step 2 Professor K next calculates the *probability* of *not* pulling an even numbered cube from the bag.

$$\text{P(not even)} = \frac{100-50}{100} = \frac{50}{100} = \frac{1}{2}$$

VOCABULARY

Odds: the chance that something will happen compared to chance that something else will happen

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Step 3 Professor K uses the formula for the *odds in favor* to calculate the odds of pulling an even numbered cube from the bag. Odds in favor of pulling an even numbered cube from the bag is calculated by dividing P(even) by P(not even). Therefore, odds in favor of pulling an even numbered cube from the bag =

$$\frac{1}{2} \div \frac{1}{2} = \frac{1}{\cancel{2}_{1}} \bullet \frac{\cancel{2}^{1}}{1} = \frac{1}{1}$$

The odds in favor of pulling an even numbered cube from the bag are *one to one*. That means there is an even chance or there is just as much chance to pull an even number as there is to pull an odd number from the bag.

In **Example Set 2** Professor K calculates the odds against pulling a cube numbered 10, 20, 30 or 40 from a bag containing 52 consecutively numbered cubes. Follow along closely to understand this formula.

Example Set 2

Determine the odds against pulling a cube numbered 10, 20, 30 or 40 from a bag containing 52 consecutively numbered cubes.

Step 1 Professor K first has to calculate the probability of pulling a cube numbered 10, 20, 30 or 40 from a bag containing 52 consecutively numbered cubes.

$$\text{P}(10, 20, 30, 40) = \frac{4}{52} = \frac{1}{13}$$

Step 2 Professor K next calculates the probability of *not* pulling a cube numbered 10, 20, 30 or 40 from a bag containing 52 consecutively numbered cubes.

$$\text{P}(\text{not } 10, 20, 30, 40) = \frac{52-4}{52} = \frac{48}{52} = \frac{12}{13}$$

Step 3 Professor K uses the formula for the odds against to calculate the odds of not pulling a cube number 10, 20, 30 or 40 from a bag containing 52 consecutively numbered cubes. Odds against pulling a cube numbered 10, 20, 30 or 40 from the bag is calculated by dividing P(not 10, 20, 30, 40) by P(10, 20, 30, 40). Odds against pulling a cube numbered 10, 20, 30 or 40 =

$$\frac{12}{13} \div \frac{1}{13} = \frac{12}{13} \bullet \frac{13}{1} = \frac{12 \bullet 13}{13 \bullet 1} = \frac{12}{1}$$

In **Example Set 3**, Professor K determines the odds of rolling certain numbers on a number cube.



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Example Set 3

A number cube is labeled with the numbers 1, 2, 3, 4, 5, and 6. Calculate the odds of rolling a number less than 6.

Step 1 You must first calculate the probability of rolling a 6.

$$P(6) = \frac{1}{6}$$

Step 2 You must now calculate the probability of rolling the numbers, 1, 2, 3, 4 or 5.

$$P(1, 2, 3, 4, 5) = \frac{5}{6}$$

Step 3 The odds in favor of rolling a number less than six is determined by dividing P(1, 2, 3, 4, 5) by P(6). Odds in favor of rolling a number less than 6 =

$$\frac{P(1,2,3,4,5)}{P(6)} = \frac{5}{6} \div \frac{1}{6} = \frac{5}{6} \bullet \frac{6}{1} = \frac{30}{6} = \frac{5}{1}$$

Therefore, the odds of rolling a number less than 6 are 5 to 1.

Show Professor K you can calculate the odds of a certain outcome by working the activities for this lesson.

LIFE PRINCIPLE



"A proper attitude will increase the odds of happiness."

—Anonymous

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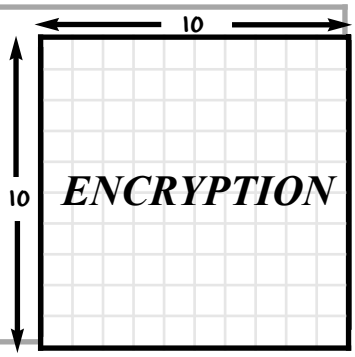
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Lesson 15

Predictions

Chapter 1



Section 3

Lessons 11-15



You have probably written things in ***code***, using numbers for letters or vice-versa. Professor K has studied many ancient civilizations that had hidden codes or meanings in their writings. Egyptians ***hieroglyphics***, Babylonian ***cuneiform*** and Hebrew writings show that they knew many of the mathematical principles that we use today.

The Egyptians estimated the value of π to at least two decimals. The Egyptians also knew much about the volume, angles and surface areas of pyramids.



Babylonian mathematics was very advanced for its time. The Babylonians plotted positions of the stars. Babylonians knew of the mathematical concept and theorem for triangles known today as the Pythagorean Theorem. The Hebrews and Greeks used letters to represent numbers, as did the Romans. You probably remember the main Roman numerals. Roman numerals are used even today in special places such as memorials and monuments. The Hebrews and Greeks used their entire alphabet to represent numbers. The Romans only used six letters to represent numbers. Professor K shows you those six letters and their ***numerical*** equivalent in **Table 15.1**. Professor K presents a quick review of how to read Roman numerals in **Example Set 1**.

VOCABULARY

Code: numbers, letters or words with hidden meanings to promote secrecy
Hieroglyphics: the name given to Egyptian "writing," composed of pictures and symbols rather than letters of an alphabet
Cuneiform: a type of writing composed of various wedge-shaped characters.
Numerical: referring to numbers
Encrypted: coded
Navajo Indians: a Native American people group in parts of the Southwest

Table 15.1

M	=	1000
D	=	500
C	=	100
L	=	50
X	=	10
V	=	5
I	=	1

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Refer to **Table 15.1** as you work through **Example Set 1** with Professor K.

Example Set 1

Determine the numerical value of these Roman numerals.

a) VI = 5 + 1 = 6; a letter of smaller numerical value to the right of a larger means addition.
b) XI = 10 + 1 = 11; a letter of smaller numerical value to the right of a larger means addition.
c) IX = 10 - 1 = 9; a letter of smaller numerical value to the left of a larger means subtraction.
d) LIX = 50 + 10 - 1 = 59; add then subtract.
e) DCCLXXIII = 500 + 100 + 100 + 50 + 10 + 10 + 1+ 1+ 1 = 773; to determine the proper numerical value you must add left to right the numerical value of each Roman letter.
f) CDXLIV = 500 - 100 + 50 - 10 + 5 - 1 = 444; remember to subtract smaller numbers to the left of larger.

To know how to read Roman numerals you must know how to break the code. The code of the writings of many ancient cultures had to be broken before the writings could be fully understood. Hidden within these ancient writings are mathematical facts that show these ancient cultures knew more than we may have thought.



King Solomon (around 950 B.C.) commissioned the construction of a pool 15 feet wide. In the Hebrew writings that speak of this pool, the value of the words involved (adding the numerical value of each letter together), shows that Solomon's builders knew the value of π.

Today, codes are used in international affairs. Intelligence agencies and armies of governments use codes to transfer information that they do not want others to know. Computers use numerical codes to produce characters and pictures on the monitor screen. Important personal information, such as bank account data, is ***encrypted*** as it passes over the Internet.

Look at **Example Set 2** as Professor K reads a coded sentence.

Example Set 2

Professor K found a paper with a series of numbers on it. It appeared to have been written by his children. Professor K determined that the numbers represented some kind of code, so he decided to break the code and see what his children had written. Below is what Professor K saw on the paper.

19 21 16 18 9 26 5 0 16 1 18 20 25 0 6 15 18 0 4 1 4 4 25

Step 1

Breaking a code is usually a series of trials-and-errors, like determining a formula for the next term in a list of numbers or objects in the *n*th layer. Professor K determined that this must be some sort of note, so he thought of what letters could the numbers represent. Professor K decided to use the count of each letter as it appears in the alphabet. (A = 1, B = 2, C = 3, D = 4, …, Z = 26) Professor K did not know what the zeros represented, but decided to proceed.

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Step 2

Professor K then replaced each number with a letter as shown.

S U P R I Z E 0 P A R T Y 0 F O R 0 D A D D Y

Professor K realized that the zeros divided words and from reading the message his children were planning a surprise birthday party. Professor K also decided he had to help them with their spelling.

Encrypted messages count on probability to help keep the code from being broken. The more complicated the code, the smaller the chance of it being broken.

During World War II, the United States asked a group of ***Navajo Indians*** to serve as "Code-Talkers" so the Japanese could not intercept and translate secret messages. For the entire length of time that the "Code-Talkers" were in service, the Japanese code-breakers never broke the code.

Show Professor K you understand numerical values of letters and words and codes by working the activities for this lesson.

Life Principle



"The things of this world cannot
be made known without a
knowledge of mathematics."

—Roger Bacon

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Intermediate Math Skills

Mathematics Charts

LENGTH - ENGLISH SYSTEM

12 inches	=	1 foot
3 feet	=	1 yard
5.5 yards	=	1 rod
320 rods	=	1 mile
1,760 yards	=	1 mile
5,280 feet	=	1 mile

VOLUME - ENGLISH SYSTEM

1 tablespoon, Tbsp, T	=	3 teaspoons, tsp
1 cup, c	=	16 tablespoons, Tbsp, T
1 pint, pt	=	2 cups, c
1 quart, qt	=	2 pints, pts
1 gallon, gal	=	4 quarts, qts

WEIGHT - ENGLISH SYSTEM

1 pound, lb	=	16 ounces, oz
1 hundredweight, cwt	=	100 pounds, lbs
1 ton	=	20 hundredweight, cwt
1 ton	=	2000 pounds, lbs

LENGTH - METRIC SYSTEM

1 meter	=	1,000 millimeters
1 meter	=	100 centimeters
1 kilometer	=	1,000 meters
1 centimeter	=	10 millimeters

VOLUME - METRIC SYSTEM

1 liter	=	1,000 milliliters
1 liter	=	100 centiliters
1 kiloliter	=	1,000 liters
1 centiliter	=	10 milliliters

WEIGHT - METRIC SYSTEM

1 gram	=	1,000 milligrams
1 gram	=	100 centigrams
1 kilogram	=	1,000 grams
1 centigram	=	10 milligrams

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PERFECT SQUARES

$\sqrt{1}=1$	$\sqrt{25}=5$	$\sqrt{81}=9$	$\sqrt{169}=13$
$\sqrt{4}=2$	$\sqrt{36}=6$	$\sqrt{100}=10$	$\sqrt{196}=14$
$\sqrt{9}=3$	$\sqrt{49}=7$	$\sqrt{121}=11$	$\sqrt{225}=15$
$\sqrt{16}=4$	$\sqrt{64}=8$	$\sqrt{144}=12$	$\sqrt{256}=16$

PERFECT CUBES

$\sqrt[3]{1}=1$	$\sqrt[3]{-8}=-2$	$\sqrt[3]{64}=4$	$\sqrt[3]{-125}=-5$
$\sqrt[3]{-1}=-1$	$\sqrt[3]{27}=3$	$\sqrt[3]{-64}=-4$	$\sqrt[3]{216}=6$
$\sqrt[3]{8}=2$	$\sqrt[3]{-27}=-3$	$\sqrt[3]{125}=5$	$\sqrt[3]{-216}=-6$

$$P = \frac{\text{the number of elements in the set containing favorable outcomes}}{\text{the number of elements in the sample set}}$$

$$Midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

FORMULAS

Surface Area	Area	Volume	Perimeter/ Circumference	Others
Sphere: $A_s = 4\pi r^2$	**Triangle:** $A = {}^1\!/_2 bh$	**Sphere:** $V = {}^4\!/_3 \pi r^3$	**Perimeter of a Rectangle:** $P = 2w + 2l$	**Pythagorean Theorem:** $c^2 = a^2 + b^2$
Cylinder: $A_s = 2\pi rh + 2\pi r^2$	**Rectangle:** $A = l \times w$	**Cone:** $V = {}^1\!/_3 \pi r^2 h$	**Circumference of a Circle:** $C = 2\pi r$ or $C = \pi d$	**Convert to Fahrenheit:** $F = {}^9\!/_5 C + 32$
Cube: $A_s = 6s^2$	**Circle:** $A = \pi r^2$	**Rectangular Prism:** $V = l \times w \times h$		**Convert to Celsius:** $C = {}^5\!/_9 (F - 32)$
Lateral Surface Area of a Cone: $A_{ls} = \pi rl$	**Trapezoid:** $A = {}^1\!/_2 (b_1 + b_2)h$	**Square Pyramid:** $V = ({}^1\!/_3) lwh$		**Distance:** $d = r \bullet t$
		Triangular Prism: $V = {}^1\!/_2 bhl$		**Slope:** $y = mx = b$
		Cylinder: $V = \pi r^2 h$		**Simple Interest:** $I = prt$

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Mathematics Charts

TIME CONVERSIONS

1 week = 7 days
1 year = 12 months

CONVERSIONS

English to Metric	
1 ounce (oz)	28.35 grams (g)
1 pound (lb)	454 grams (g)
1 ton (T)	907.2 kilograms (kg)

Metric to English	
1 gram (g)	.035 ounce (oz)
1 kilogram (kg)	2.2 pounds (lb)
1 metric ton (mt)	2200 pounds (lb)

English to Metric	
1 cubic inch	16.39 cubic cm
1 gallon	3.785 liters
1 cubic foot	28.316 liters
1 quart	.946 liters

Metric to English	
1 cubic cm	.061 cubic inch
1 cubic meter	35.315 cubic feet
1 liter	1.05 quarts
1 liter	.264 gallon

English to Metric	
1 inch	2.54 cm
1 foot	0.3048 meter
1 yard	0.9144 meter
1 mile	1.6 km

Metric to English	
1 centimeter	0.3937 in.
1 meter	39.37 in.
1 meter	1.09 yds
1 kilometer	0.6213 mile

AREA CONVERSIONS

1 square centimeter = 0.155000 sq in.
1 square meter = 10.763910 sq ft
1 square inch = 645.16 sq mm
1 square foot = 0.09290304 sq meter
1 square yard = 0.836127 sq meter
1 square foot = 144 sq in.

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